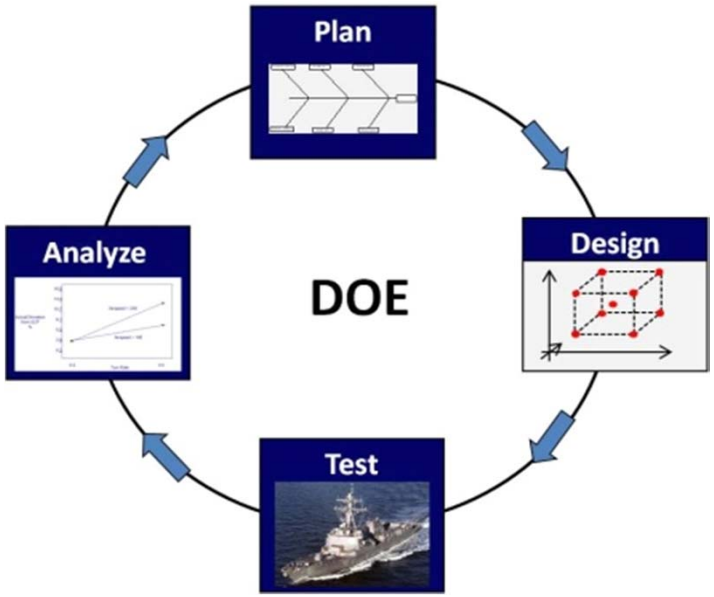
Experimental Designs

Experimental Design Overview

- **Experimental Design Types**
  - Factorial designs and fractional factorial designs
  - Response surface methodology
  - Optimal designs
  - Restricted randomization designs
    - » Blocking
    - » Split-plots
- **Design Evaluation**
  - Statistical measures of merit
- **Designs for Software Testing**

Steps in Designing an Experiment

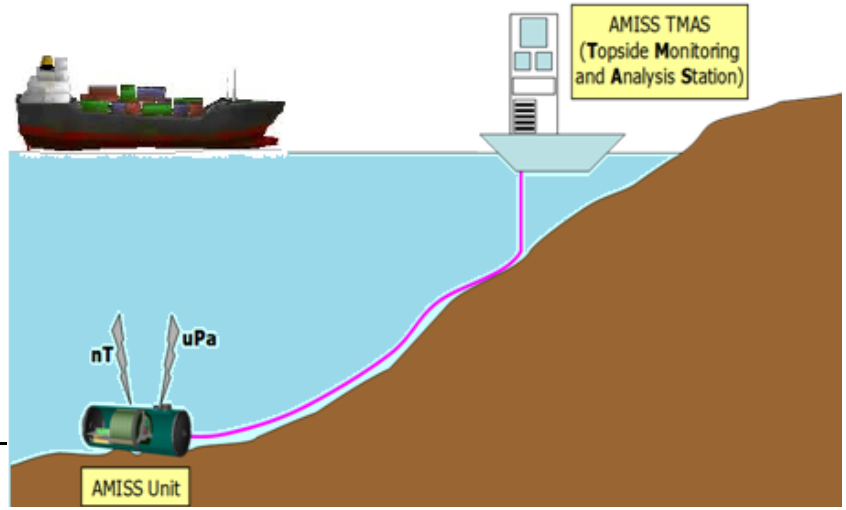
1. Define the objective of the experiment
2. Select appropriate response variables
3. Choose factors, levels
4. **Choose experimental design**
   - Disallowed combinations of factors (safety, operational realism)
   - Realistic range for test resources
   - Allowable test risk
   - Analysis objectives
5. Perform the test
6. Statistically analyze the data
7. Draw conclusions



Steps are strategically linked into a defensible process.

Motivating Example: Test Plan for Mine Susceptibility

- **Goal:**
  - Develop an adequate test to assess the susceptibility of a cargo ship against a variety of mine types using the Advanced Mine Simulation System (AMISS).
- **Responses:**
  - Magnetic signature, acoustic signature, pressure
  - Slant range at simulated detonation
- **Factors:**
  - Speed, range, degaussing system status
- **Other considerations:**
  - Water depth
  - Ship direction



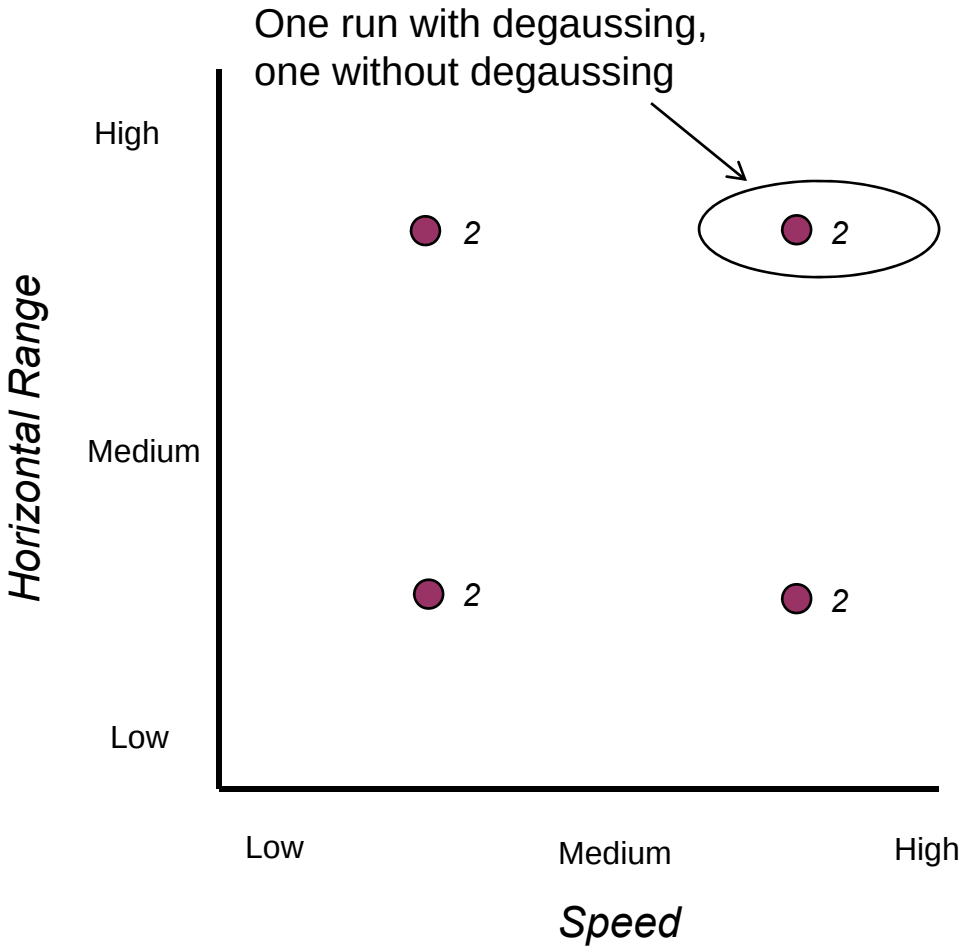
Test Designs

Design Type	Number of Runs
Full Factorial (2-level)	8
Fractional Factorial Design	4
Full Factorial Design (with center point)	10
Full Factorial (2-level) replicated	16
General Factorial (3x3x2)	18
Response Surface Design: Central Composite Design	18
Central Composite Design (replicated center point)	20
Central composite Design with replicated factorial points (Large CCD)	28
Optimal Design	Varies with model selected

Test Designs

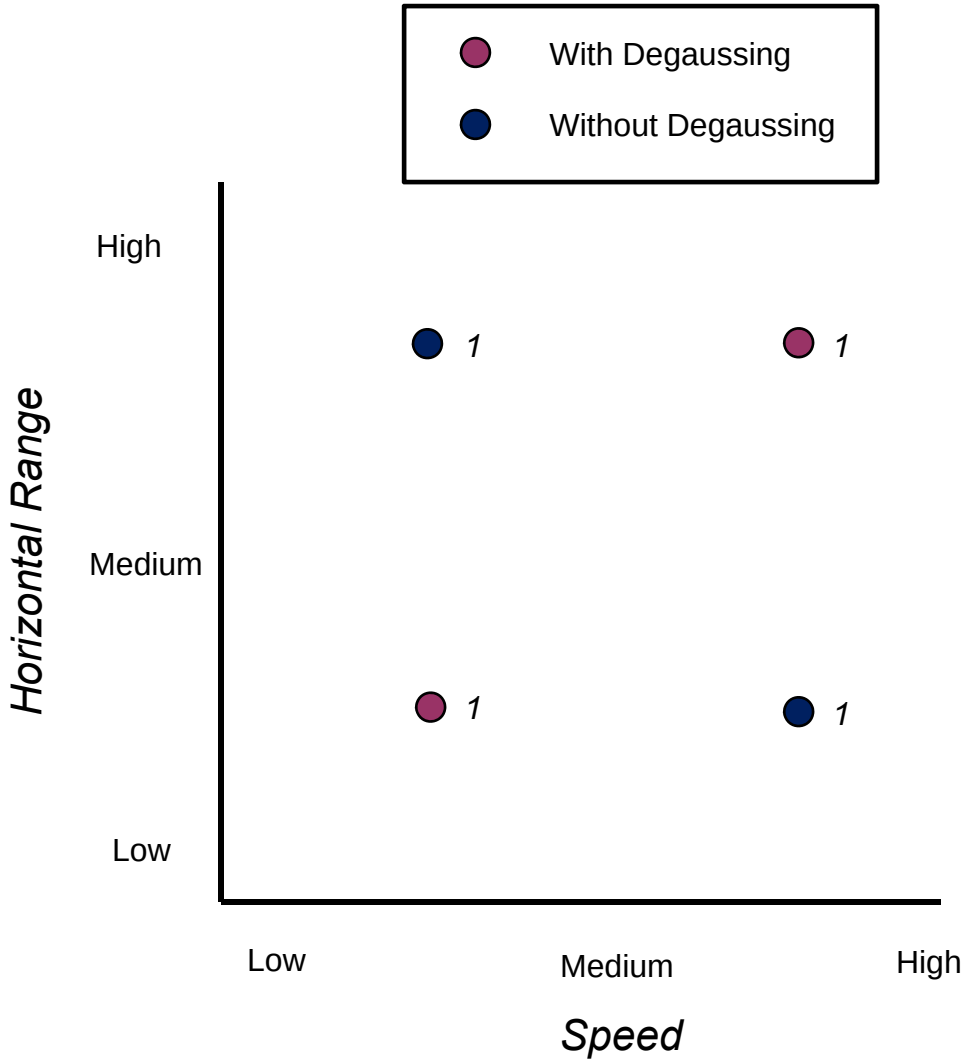
Full Factorial Designs (2 – level)

- A design with two or more factors, each with two levels, where all possible factor combinations are tested at least once.
- Typically used in DT and OT when the total number of factors and factor combinations is not too large (e.g., 3-5 factors).
- A full factorial design allows for the estimation of all main effects and interaction terms in the model.
- Full factorial designs tend to provide too much information (over powered) for large numbers of factors.



Test Designs

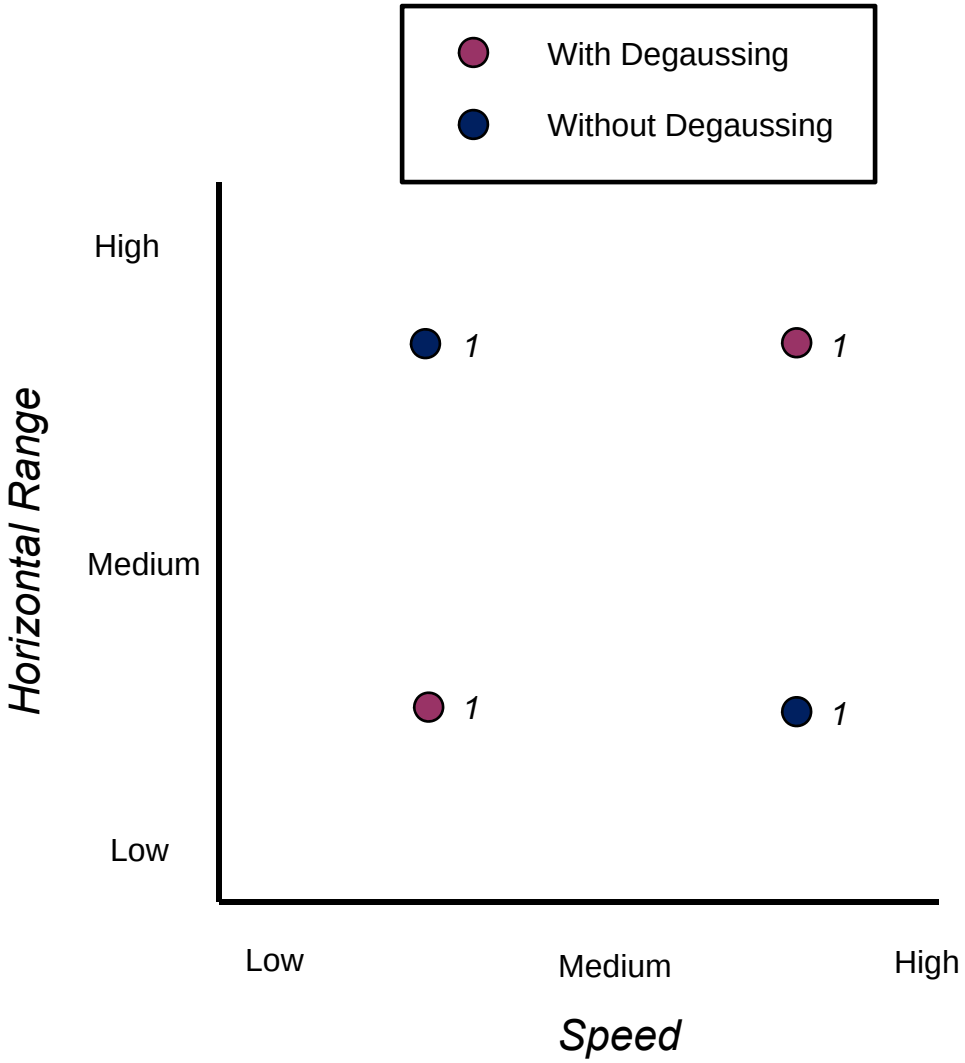
Design Type	Number of Runs
Full Factorial (2-level)	8
Fractional Factorial Design	4
Full Factorial Design (with center point)	10
Full Factorial (2-level) replicated	16
General Factorial (3x3x2)	18
Response Surface Design: Central Composite Design	18
Central Composite Design (replicated center point)	20
Central composite Design with replicated factorial points (Large CCD)	28
Optimal Design	Varies with model selected



Test Designs

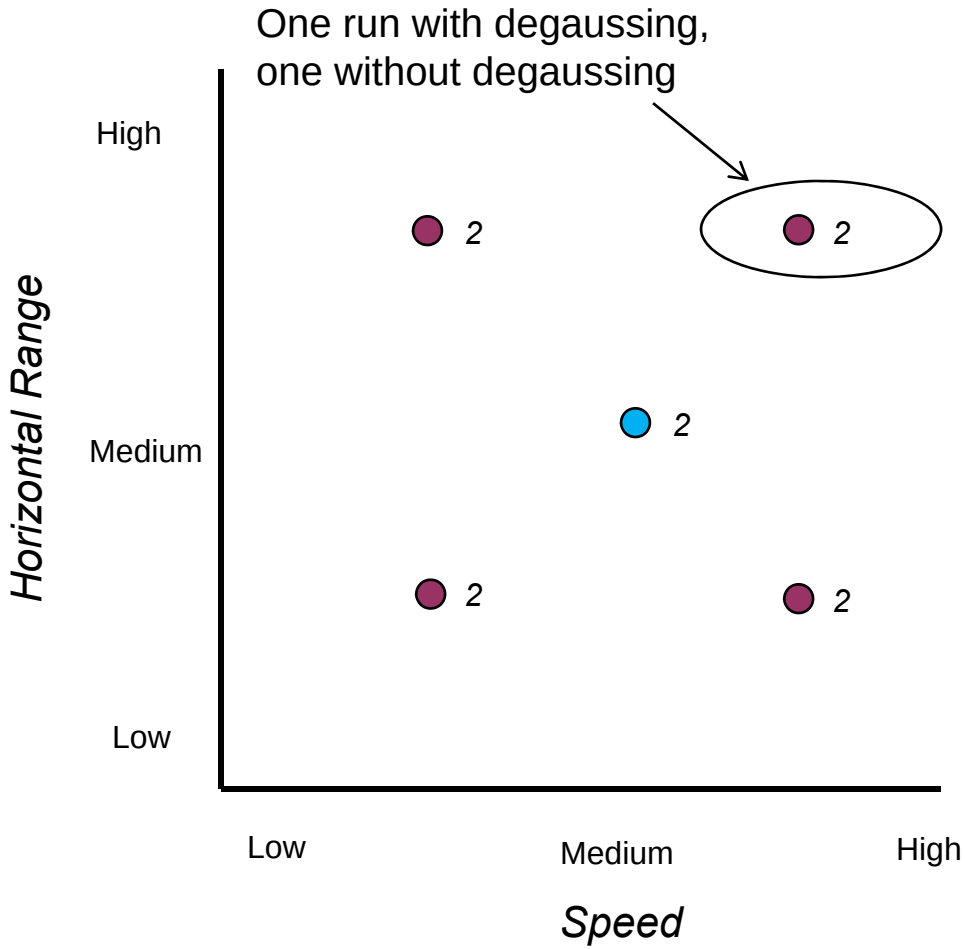
Fractional Factorial Designs

- A fractional factorial design consists of a strategically selected subset of runs from a full factorial design
- Useful when:
  - Large number of factors and it is uneconomical to test every possible factor combination
  - In screening experiments to identify the primary factors
- Typically, fractional factorial designs that allow for two-way interactions are adequate to characterize system performance
  - Leverages sparsity of effects: most systems are dominated by some of the main effects and low order interactions



Test Designs

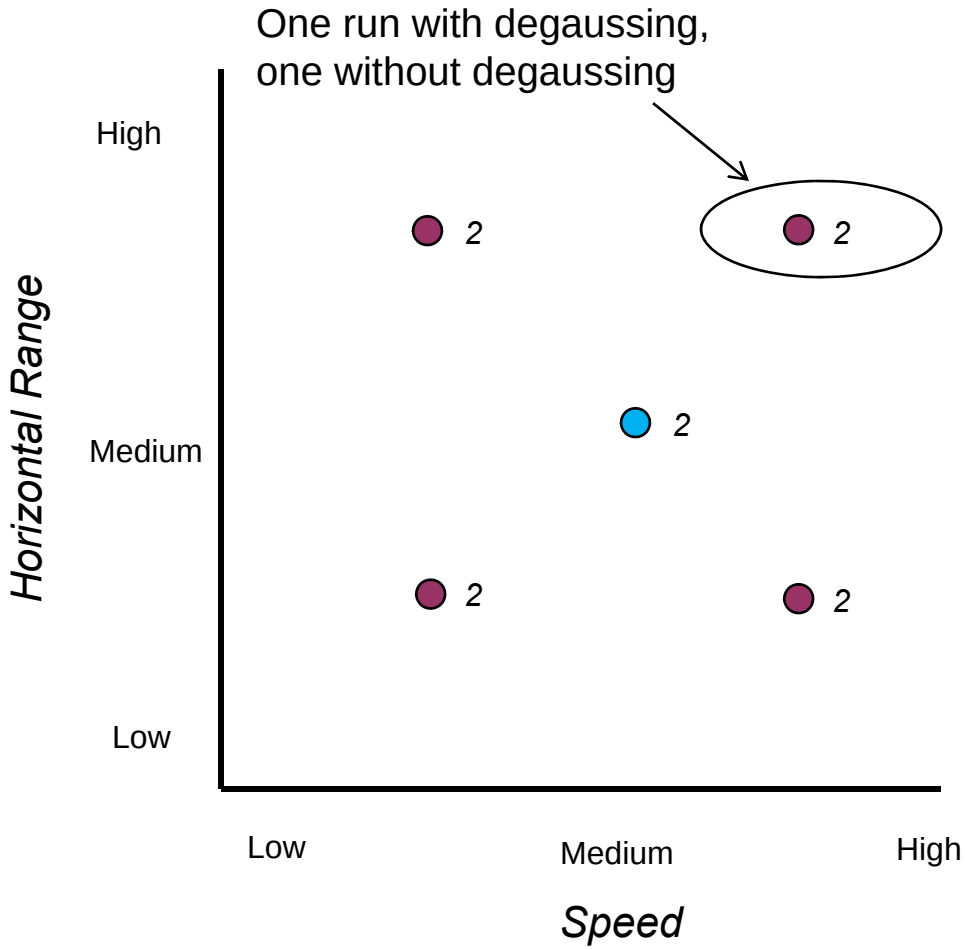
Design Type	Number of Runs
Full Factorial (2-level)	8
Fractional Factorial Design	4
Full Factorial Design (with center point)	10
Full Factorial (2-level) replicated	16
General Factorial (3x3x2)	18
Response Surface Design: Central Composite Design	18
Central Composite Design (replicated center point)	20
Central composite Design with replicated factorial points (Large CCD)	28
Optimal Design	Varies with model selected



Test Designs

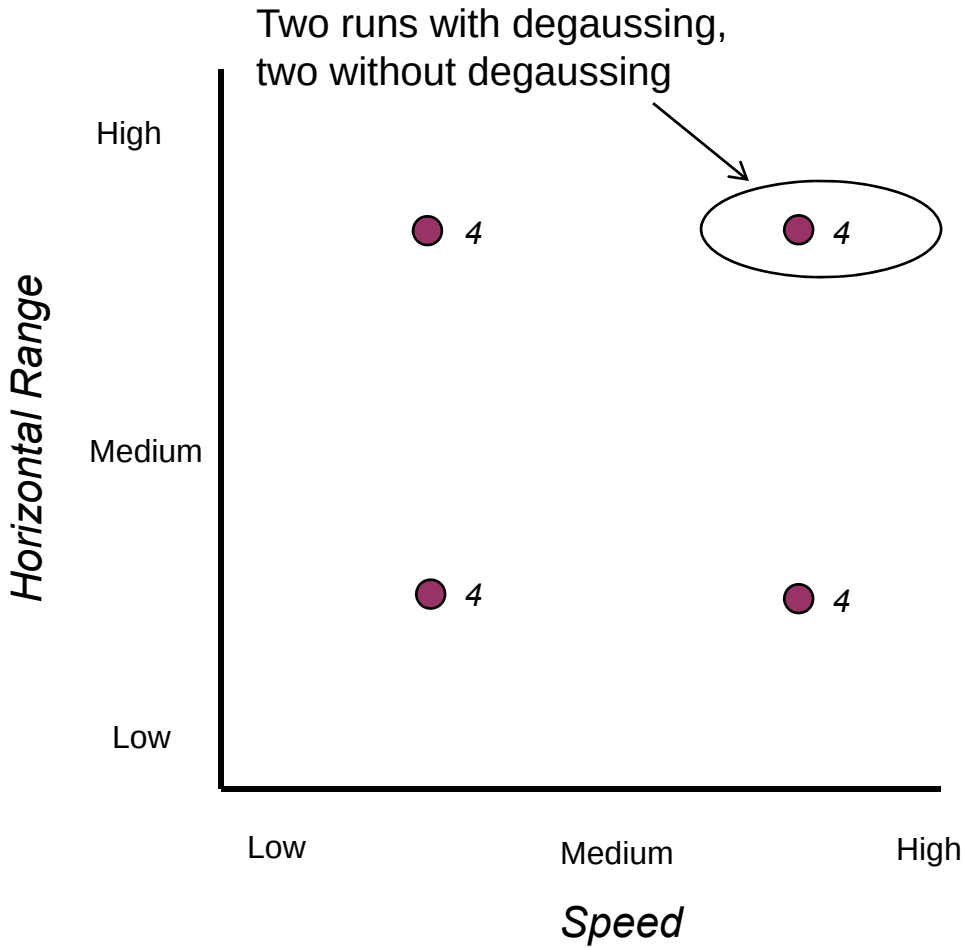
Center Points

- Add the ability to check for curvature across continuous factors
- Provide small increases to statistical power



Test Designs

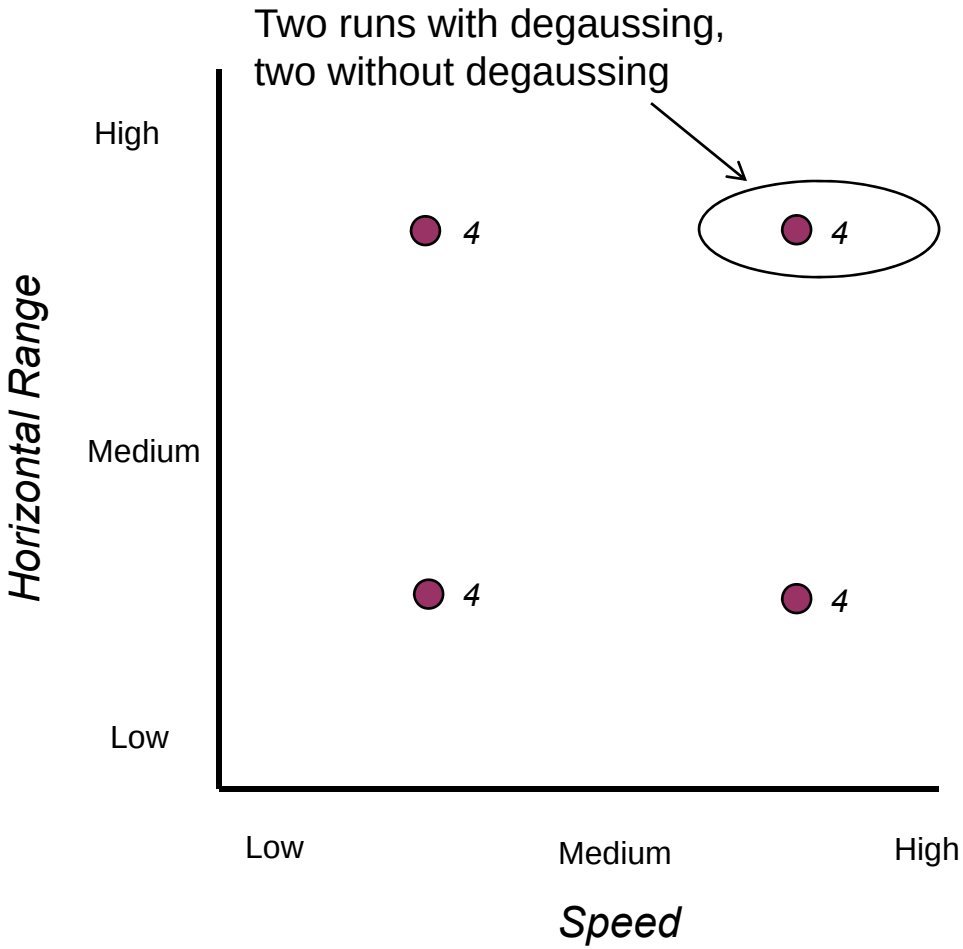
Design Type	Number of Runs
Full Factorial (2-level)	8
Fractional Factorial Design	4
Full Factorial Design (with center point)	10
Full Factorial (2-level) replicated	16
General Factorial (3x3x2)	18
Response Surface Design: Central Composite Design	18
Central Composite Design (replicated center point)	20
Central composite Design with replicated factorial points (Large CCD)	28
Optimal Design	Varies with model selected



Test Designs

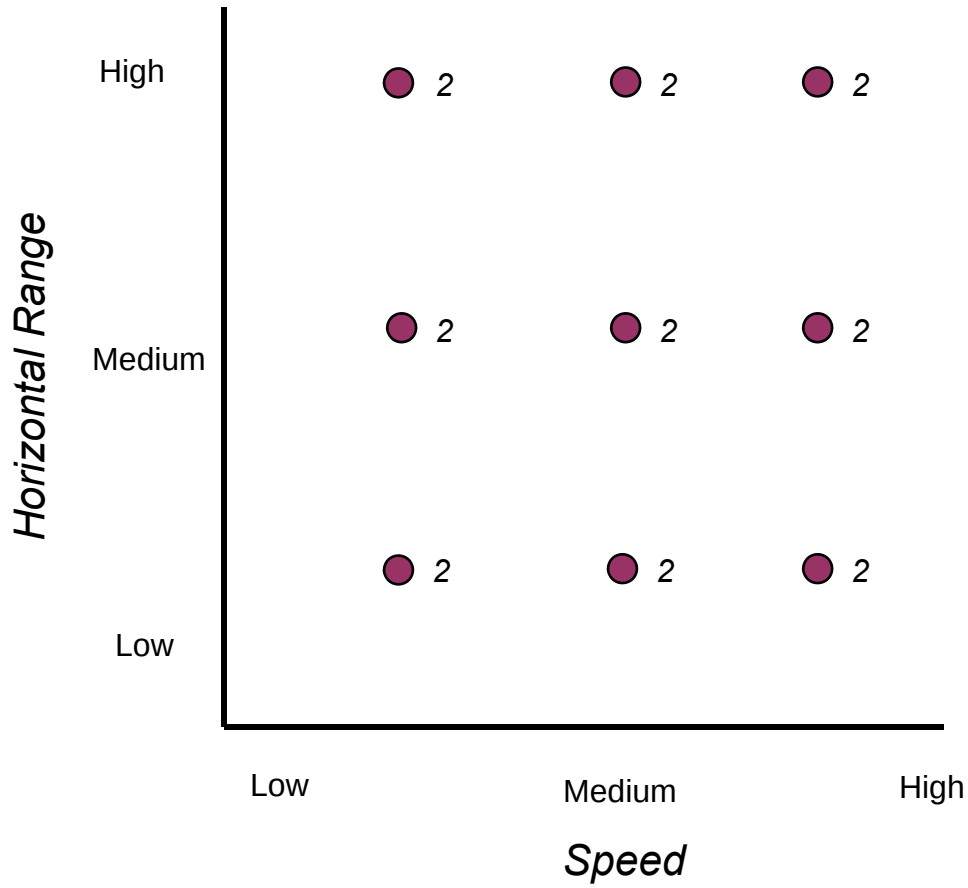
Replication

- Can be used to increase statistical power
- Provide estimates of variation within a condition
- Often not possible in cost constrained operational tests
- In a constrained resource environment it is better to cover more of the operational space than to replicate (i.e., do not eliminate a factor for the sake of replication)
- A common middle ground is to only replicate a subset of the design (e.g., a center point)



Test Designs

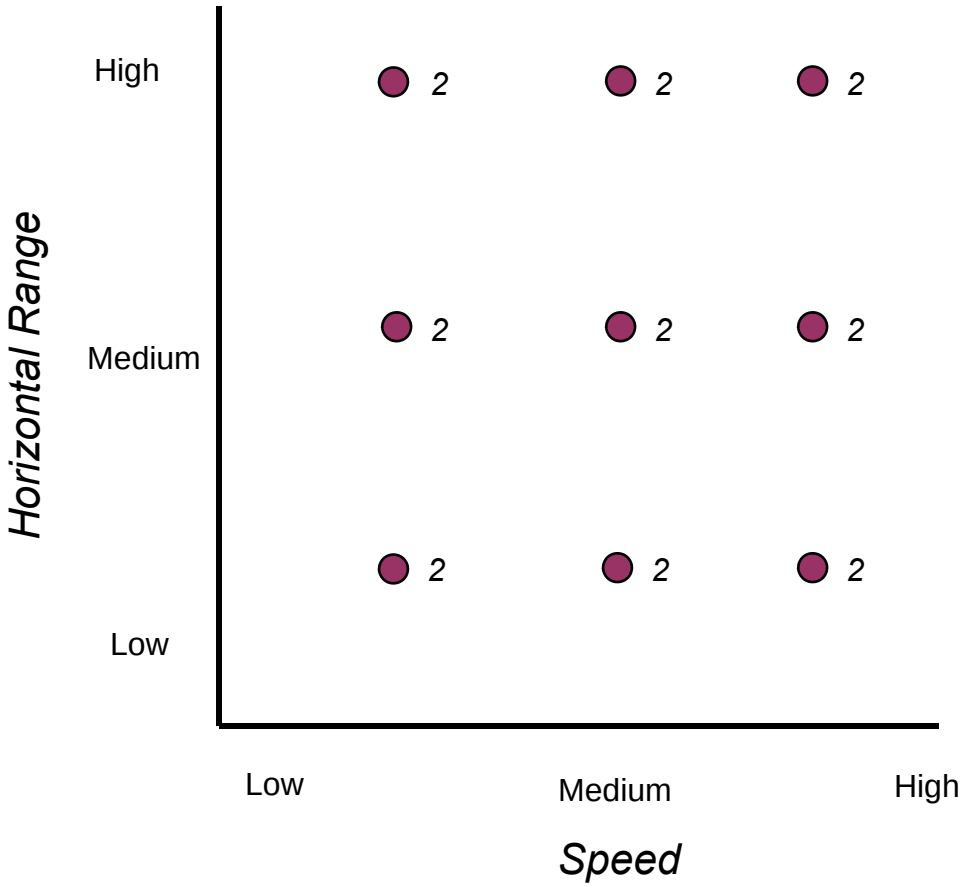
Design Type	Number of Runs
Full Factorial (2-level)	8
Fractional Factorial Design	4
Full Factorial Design (with center point)	10
Full Factorial (2-level) replicated	16
General Factorial (3x3x2)	18
Response Surface Design: Central Composite Design	18
Central Composite Design (replicated center point)	20
Central composite Design with replicated factorial points (Large CCD)	28
Optimal Design	Varies with model selected



Test Designs

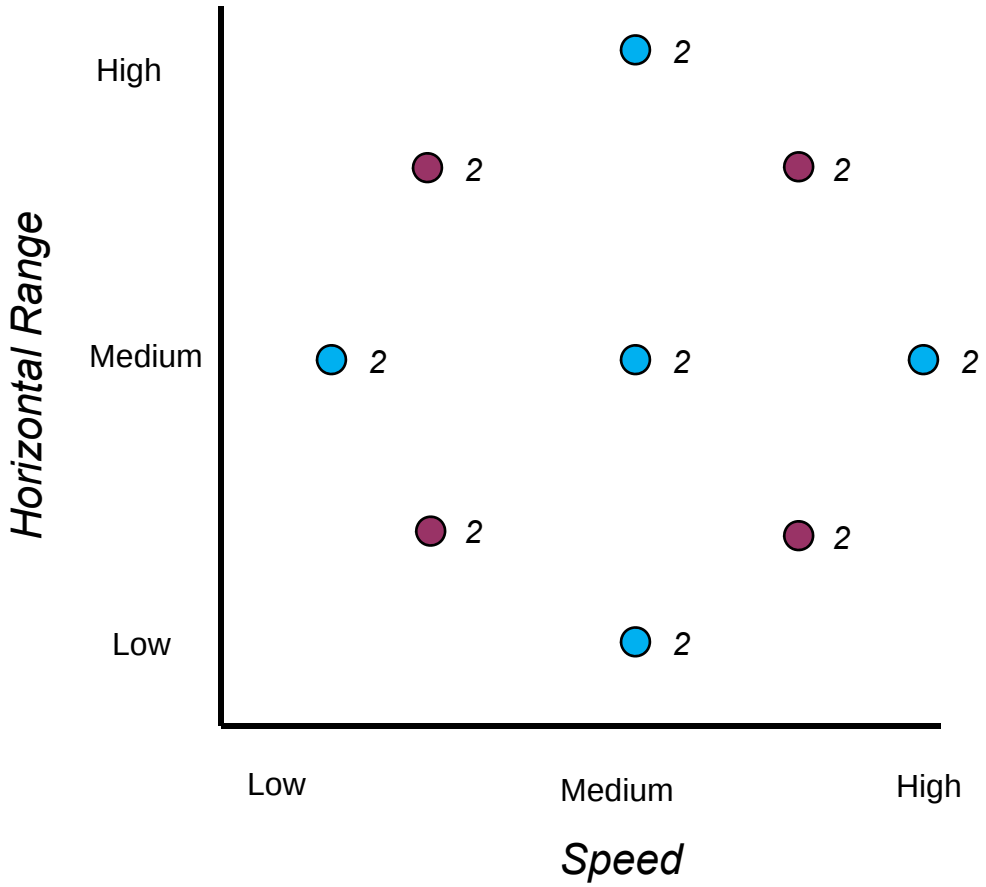
General Factorial Designs

- Similar to a two-level factorial design, designs with two or more factors, each with two or more levels, where all possible factor combinations are tested at least once.
- Only possible when the number of factors is not too large (e.g., 3-5 factors).
- Allows for the estimation of all main effects and interaction terms in the model.
- Less powerful as you add more levels to each factor
  - For continuous factors, two-levels provides the highest power



Test Designs

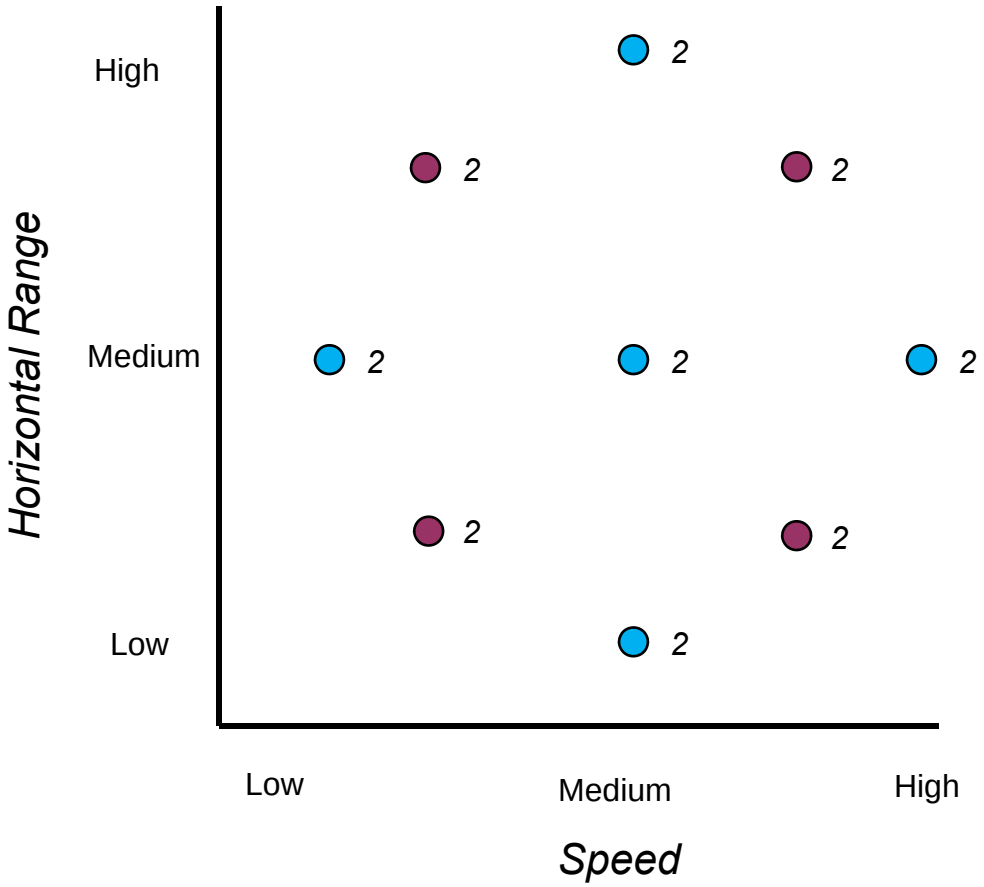
Design Type	Number of Runs
Full Factorial (2-level)	8
Fractional Factorial Design	4
Full Factorial Design (with center point)	10
Full Factorial (2-level) replicated	16
General Factorial (3x3x2)	18
Response Surface Design: Central Composite Design	18
Central Composite Design (replicated center point)	20
Central composite Design with replicated factorial points (Large CCD)	28
Optimal Design	Varies with model selected



Test Designs

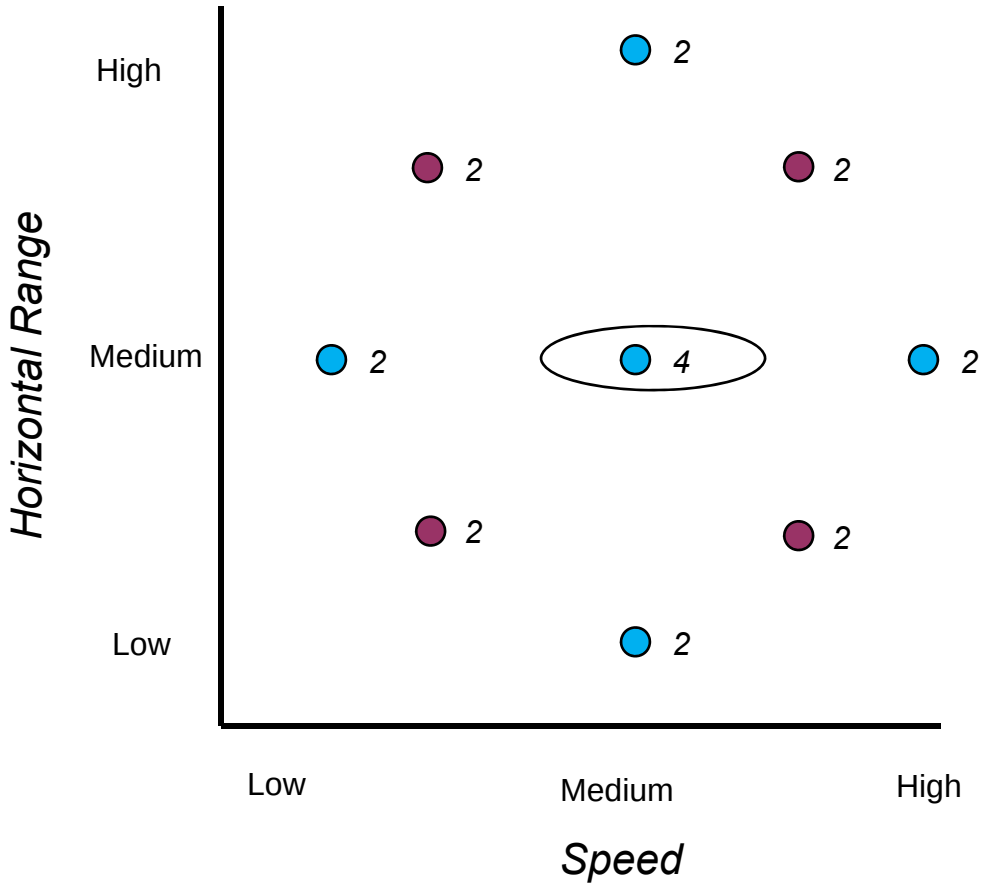
Response Surface Designs

- **Response Surface Methodology is a collection of experimental designs**
  - Originally invented by the chemical industry to conduct sequential experimentation for process optimization
  - Evolved to be a broad class of designs that characterize system performance
  - Robust test design methodology fits second order models including quadratic effects for flexible performance characterization
- **Types of Response Surface Designs:**
  - Central Composite Design, Face Centered Cube Design, Small Central Composite Design, Box-Behnken Designs, Optimal Designs



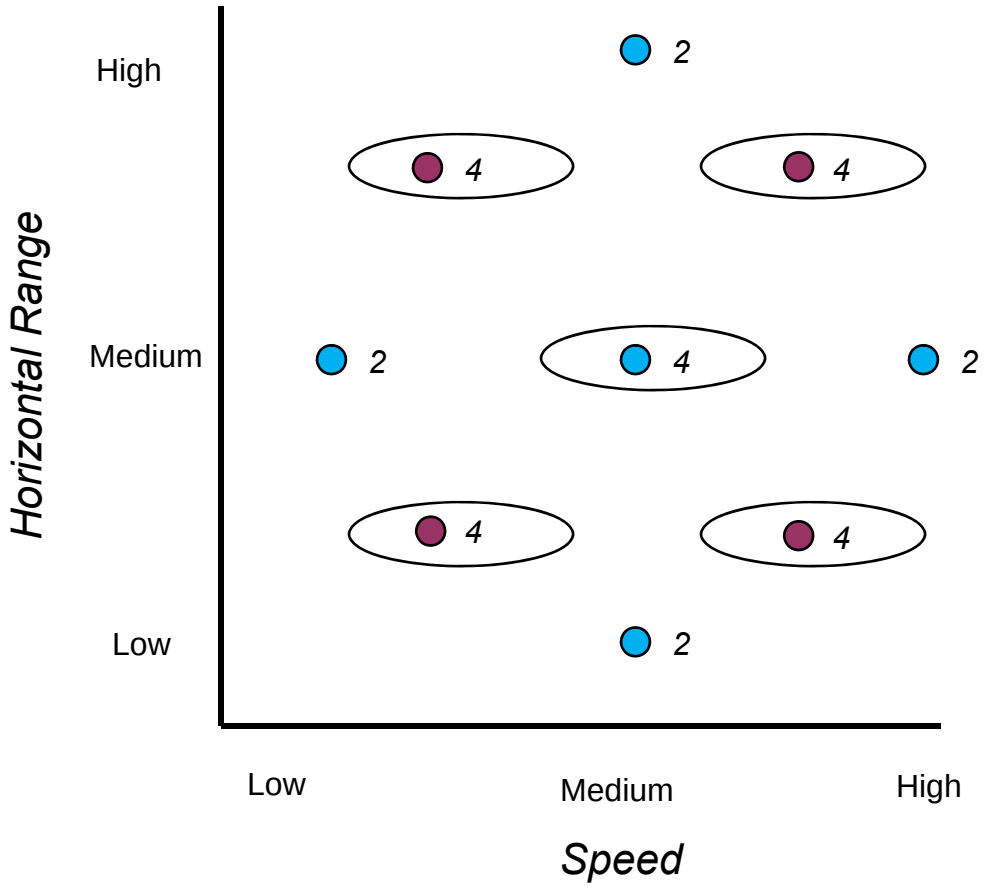
Test Designs

Design Type	Number of Runs
Full Factorial (2-level)	8
Fractional Factorial Design	4
Full Factorial Design (with center point)	10
Full Factorial (2-level) replicated	16
General Factorial (3x3x2)	18
Response Surface Design: Central Composite Design	18
Central Composite Design (replicated center point)	20
Central composite Design with replicated factorial points (Large CCD)	28
Optimal Design	Varies with model selected



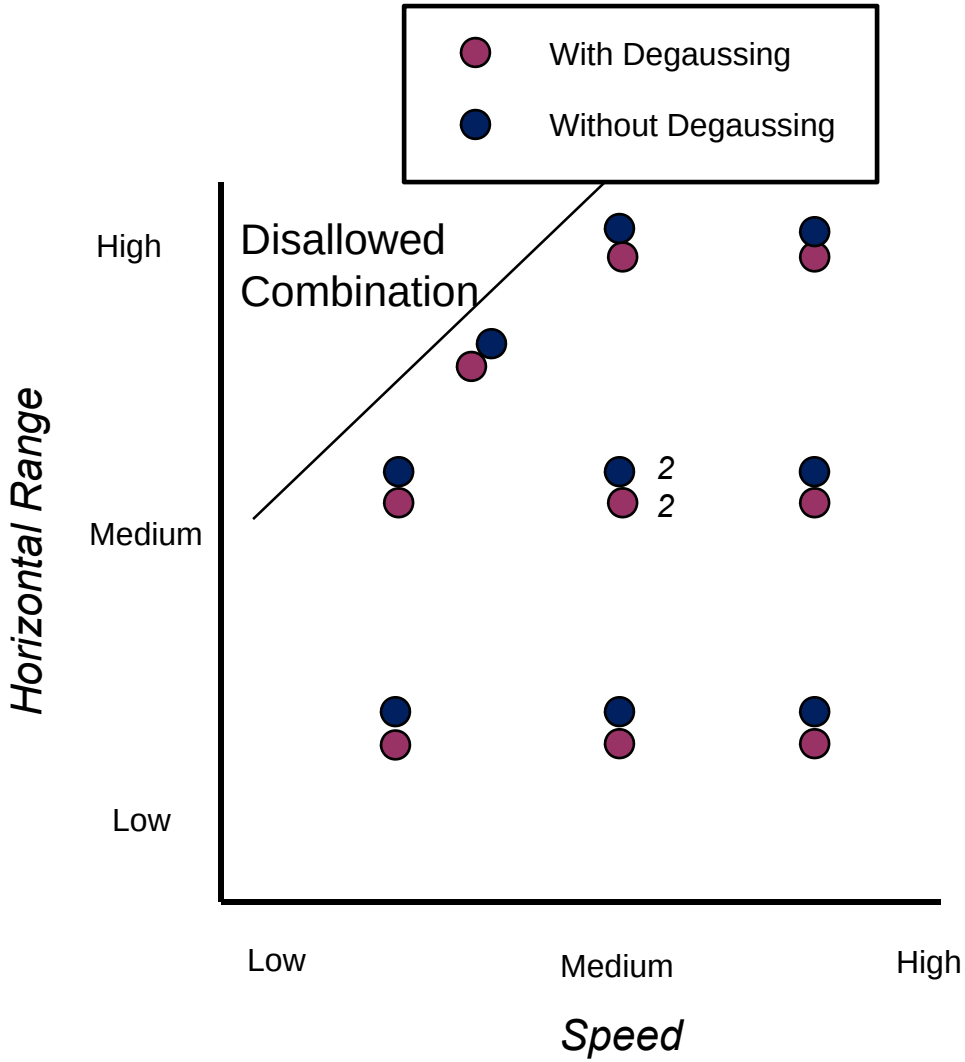
Test Designs

Design Type	Number of Runs
Full Factorial (2-level)	8
Fractional Factorial Design	4
Full Factorial Design (with center point)	10
Full Factorial (2-level) replicated	16
General Factorial (3x3x2)	18
Response Surface Design: Central Composite Design	18
Central Composite Design (replicated center point)	20
Central composite Design with replicated factorial points (Large CCD)	28
Optimal Design	Varies with model selected



Test Designs

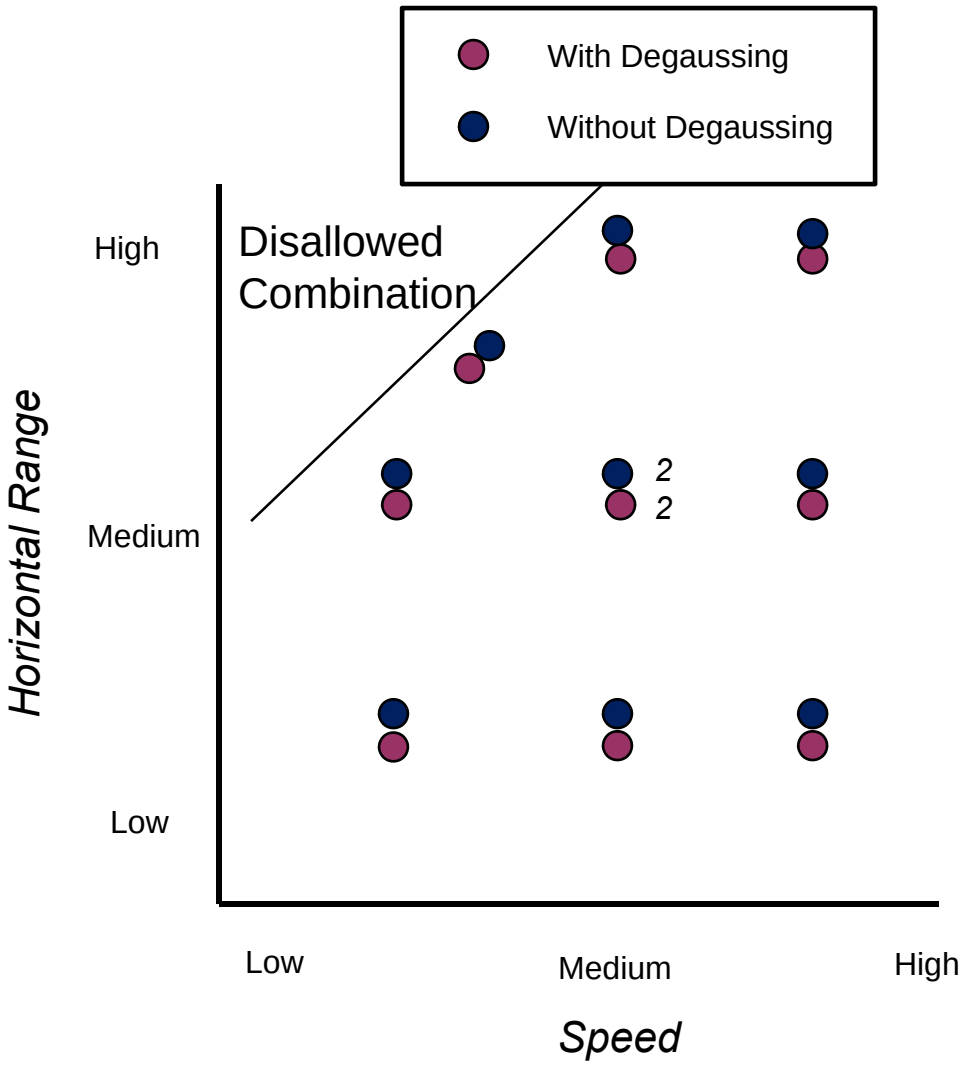
Design Type	Number of Runs
Full Factorial (2-level)	8
Fractional Factorial Design	4
Full Factorial Design (with center point)	10
Full Factorial (2-level) replicated	16
General Factorial (3x3x2)	18
Response Surface Design: Central Composite Design	18
Central Composite Design (replicated center point)	20
Central composite Design with replicated factorial points (Large CCD)	28
Optimal Design	Varies with model selected



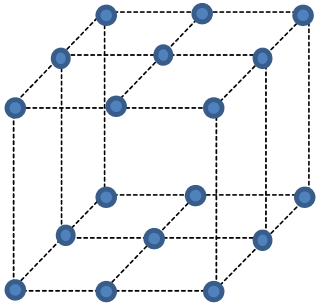
Test Designs

Optimal Designs

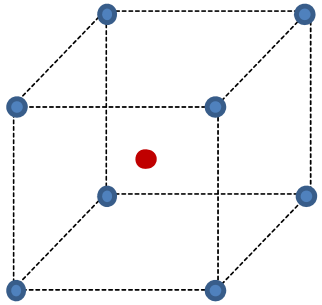
- **Optimize the test points for a known analysis model and sample size**
- **Optimal designs are useful:**
  - Large number of factors
  - Highly constrained design region (disallowed combinations of factors)
  - Large number of categorical factors
- **The optimal design fallacy**
  - Designs that are optimal under one criteria might be far from optimal under another criteria
- **Optimal designs are similar to factorial designs and response surface designs for similar analysis models**
- **Always build in extra points to optimal designs to allow for incorrect model assumptions and statistical power**



A Structured Approach to Picking Test Points
(Tied to Test Objectives and Connected to the Anticipated Analysis!)

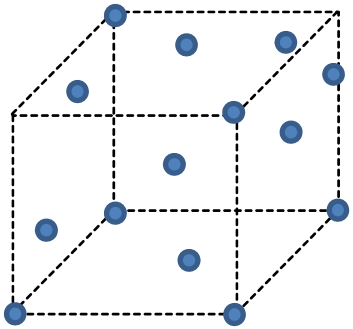


General Factorial
3x3x2 design

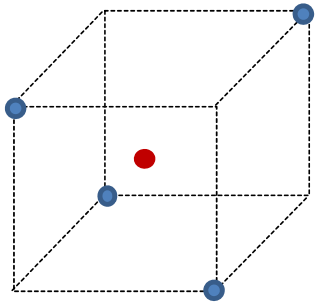


2-level Factorial
2^3 design

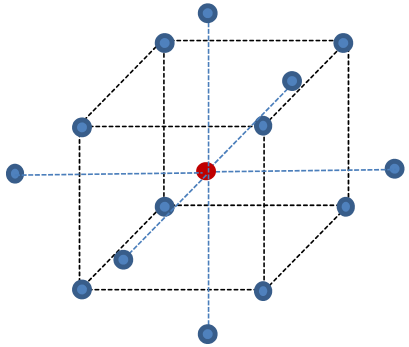
"Just Enough" test points: – most efficient



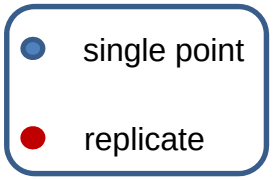
Optimal Design
IV-optimal



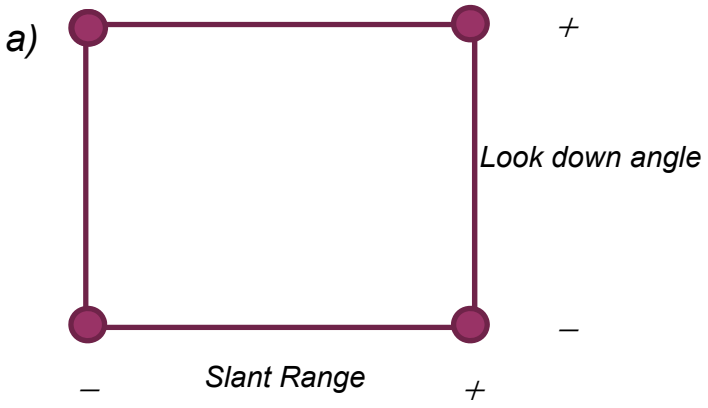
Fractional Factorial
2^{3-1} design

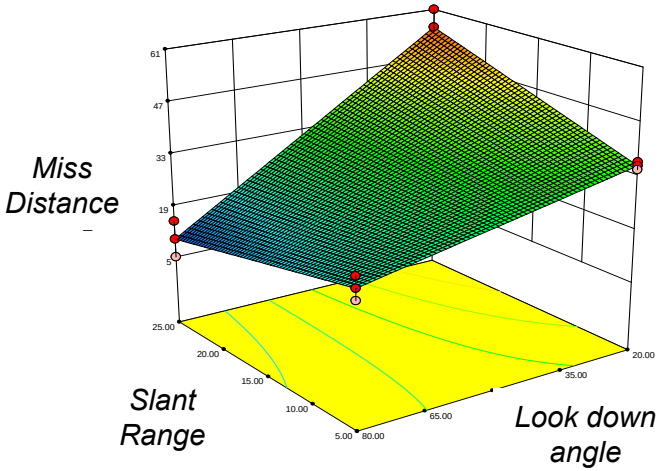


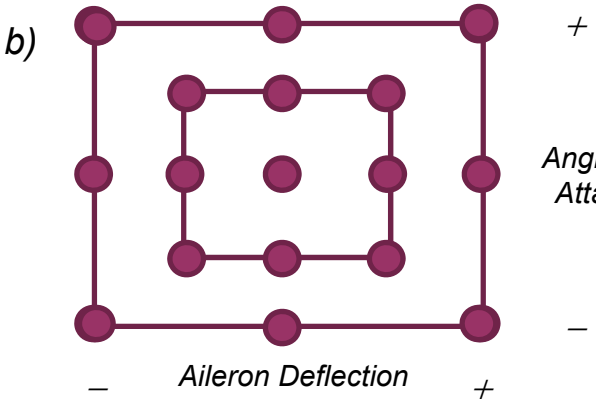
Response Surface
Central Composite design

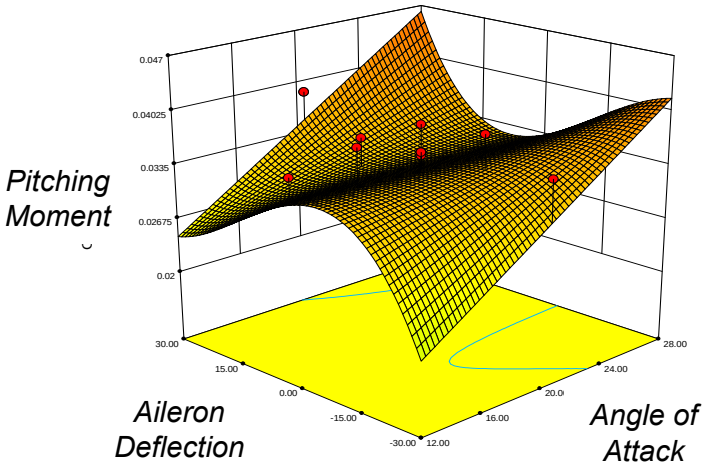


Test Design Supports the Model (The Analysis we expect to perform)









A Quick Summary: Restricted Randomization Designs

- **Randomization is a fundamental design principle**
  - Allows for mathematics that makes statistical models valid
- **Often in testing it is very expensive or impossible to completely randomize a test design**
- **Two important developments in DOE:**
  - Blocking: a design technique used to improve precision in the results
    - » Focuses on eliminating variability cause by uncontrollable factors
    - » Key aspect: we lose our ability to test for the effect of the block
    - » Example: sea trials for a surface ship one might consider blocking by location
  - Split-Plots: a design technique used when there are hard to change factors present but we still wish to estimate the effect of the hard -to-change factor.
  - Key difference between blocking and split-plot designs:
    - » Do we need to be able to determine the cause of performance differences across the factor levels?

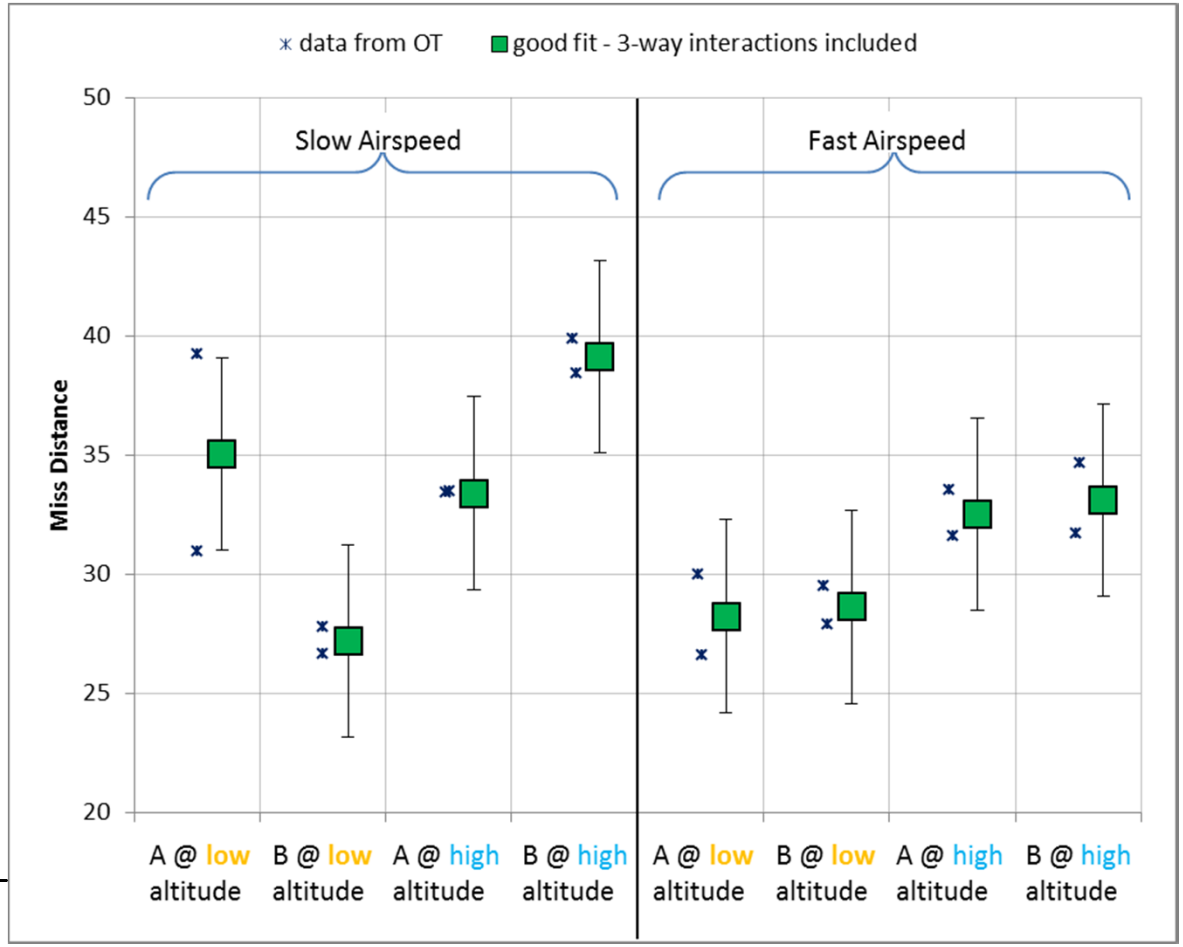
Assessing the Adequacy of Test Designs: Statistical Measures of Merit

Statistical Measure of Merit	Experimental Design Utility	Usage
Statistical Model Supported (Model Resolution/Strength)	Describes the flexibility of the empirical modeling that is possible with the test design	Match to the design goal, and expected physical response of the system. (Second order is normally adequate for characterization.)
Confidence	Quantifies the likelihood in concluding a factor has no effect on the response variable when it really has no affect.	Maximize
Power	Quantifies the likelihood in concluding a factor has an effect on the response variable when it really does.	Maximize
Correlation Coefficients	Describes degree of linear relationship between individual factors.	Minimize correlation between factors
Variance Inflation Factor	A one number summary describing the degree of collinearity with other factors in the model (provides less detail then the individual correlation coefficients).	1.0 is ideal, aim for less than 5.0
Scaled Prediction Variance	Gives the variance (i.e., precision) of the model prediction at a specified location in the design space (operational envelope).	Balance over regions of interest
Fraction of Design Space	Summarizes the scaled prediction variance across the entire design space (operational envelope).	Keep close to constant (horizontal line) for a large fraction of the design space
Optimality Criteria	Provides rank ordering of designs based on individual optimality criteria	Useful for comparing between optimal designs

Statistical Model Supported

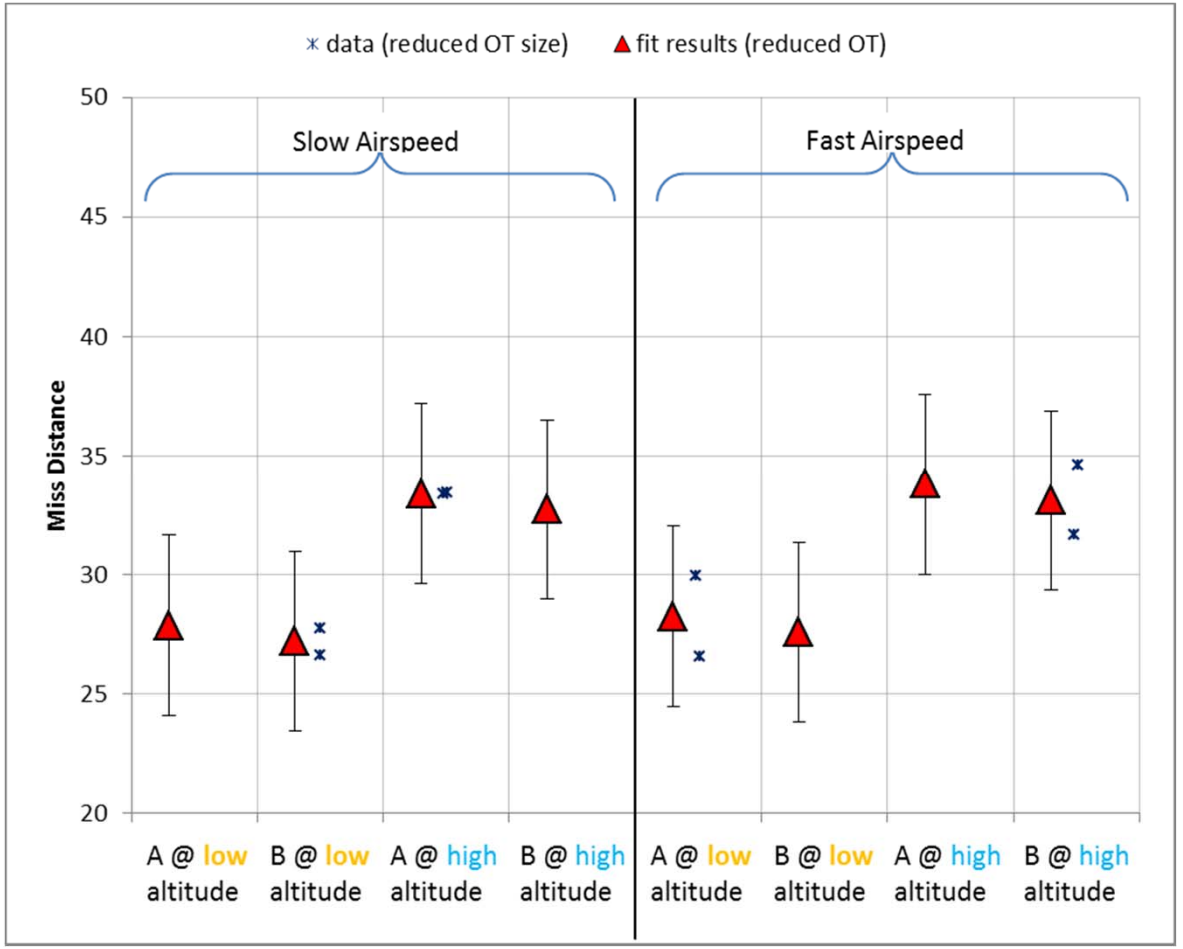
- **The type of model supported by the design is the <u>most important</u> statistical consideration when assessing test adequacy**
- **Example: Miss distance for a new missile**
  - Three two-level factors: Air Speed, Altitude, Variant (two, A and B)

- **<u>Good test planning</u>: We anticipated the need for a higher-order model, and we planned a test to capture important *interactions***
- **Model fit: 3rd order (three-way interactions)**
  - Analysis accurately reflects data under all conditions



Statistical Model Supported: 1st Order Fit

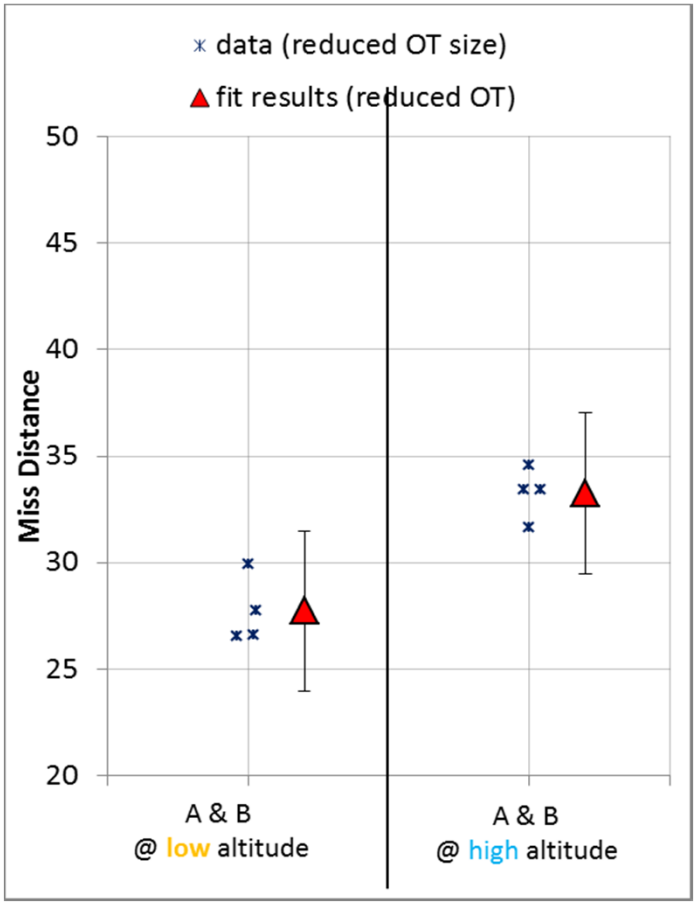
- **If we had not anticipated the need to a higher-order model, we might have planned a much smaller test**
  - Fractional-factorial only requires 8 events
- **Model fit: best we can do is a 1st-order (main-effects only) model**



  - Conclusions:
    - » Airspeed is not significant
    - » A and B are performing similarly

Statistical Model Supported: 1st Order Fit

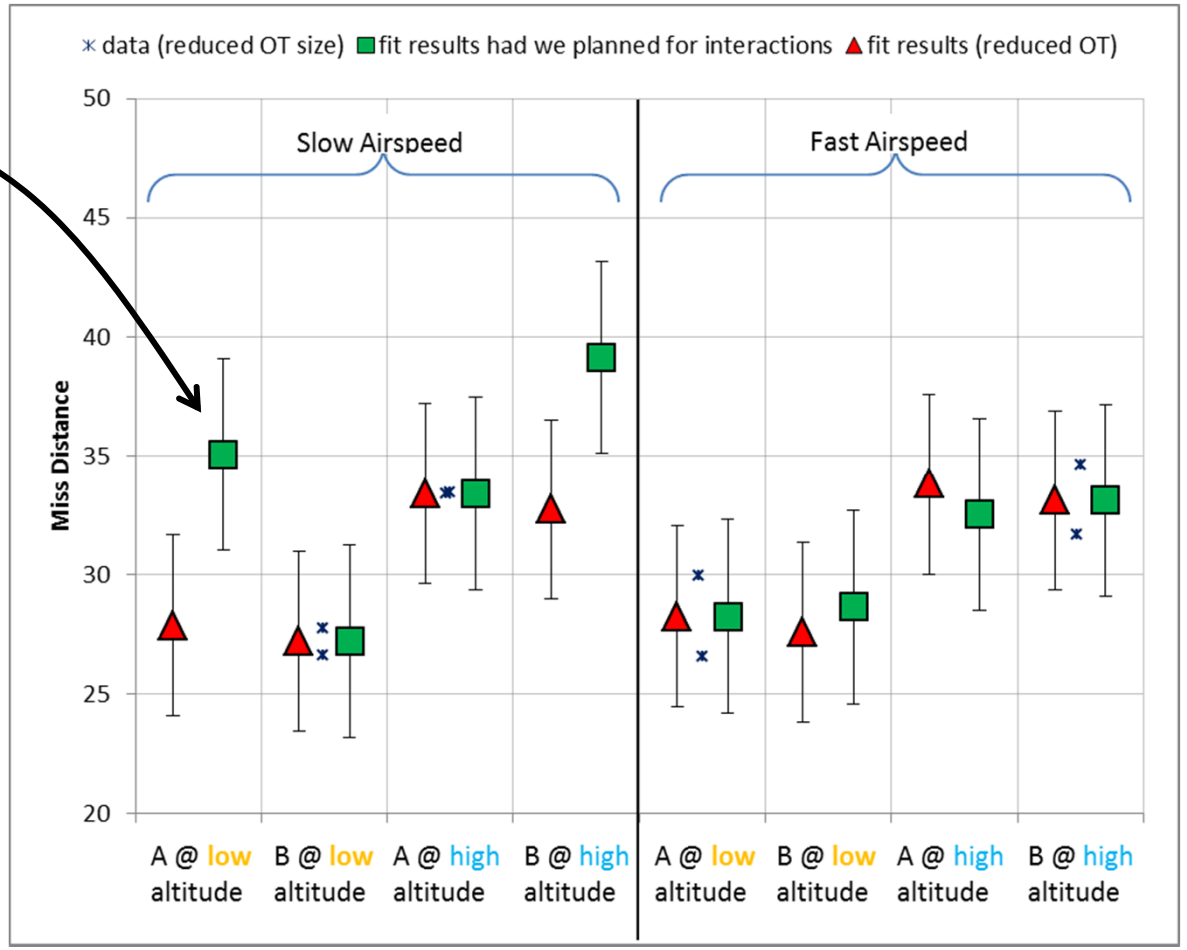
- **If we had not anticipated the need to a higher-order model, we would have planned a much smaller test**
  - Fractional-factorial only requires 8 events
- **Model fit: best we can do is a 1st-order (main-effects only) model**



  - Conclusions:
    - » Airspeed is not significant
    - » A and B are performing similarly

Statistical Model Supported: what we missed…

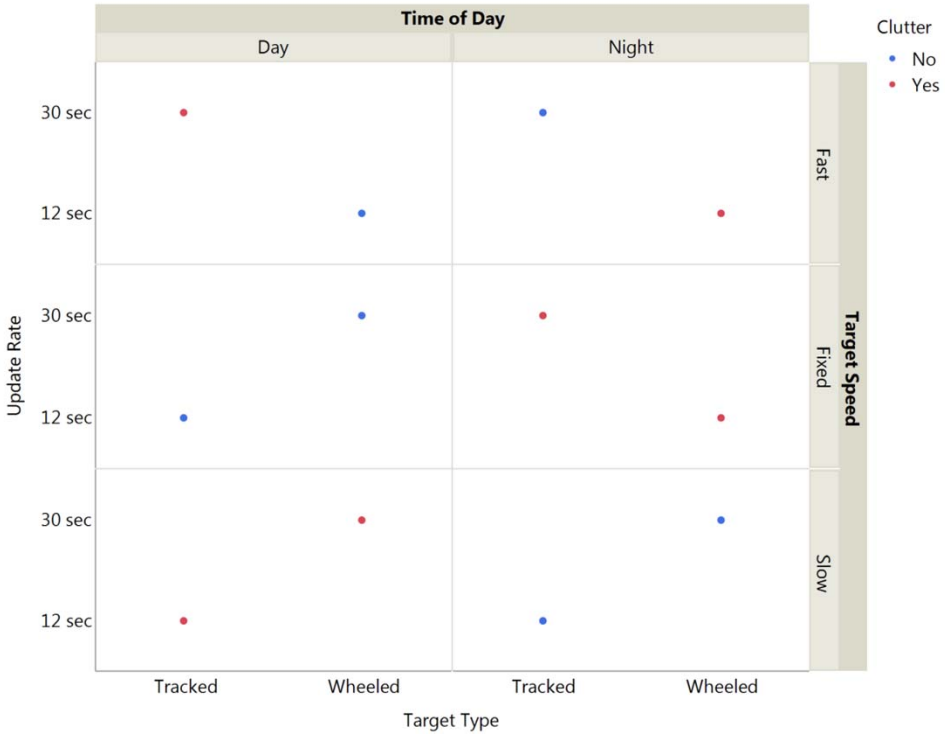
- **We missed an important 3-way interaction**
  - Variant A @ low altitude and slow airspeed performed poorly
  - BLRIP would have erroneously concluded performance was good
- **Interestingly, the lower-order model (and reduced OT size) was sufficient to capture performance for all fast airspeed conditions**
  - Results from lower order models may be accurate when there are no interactions.



Test Planning must carefully consider the analysis we anticipate conducting!

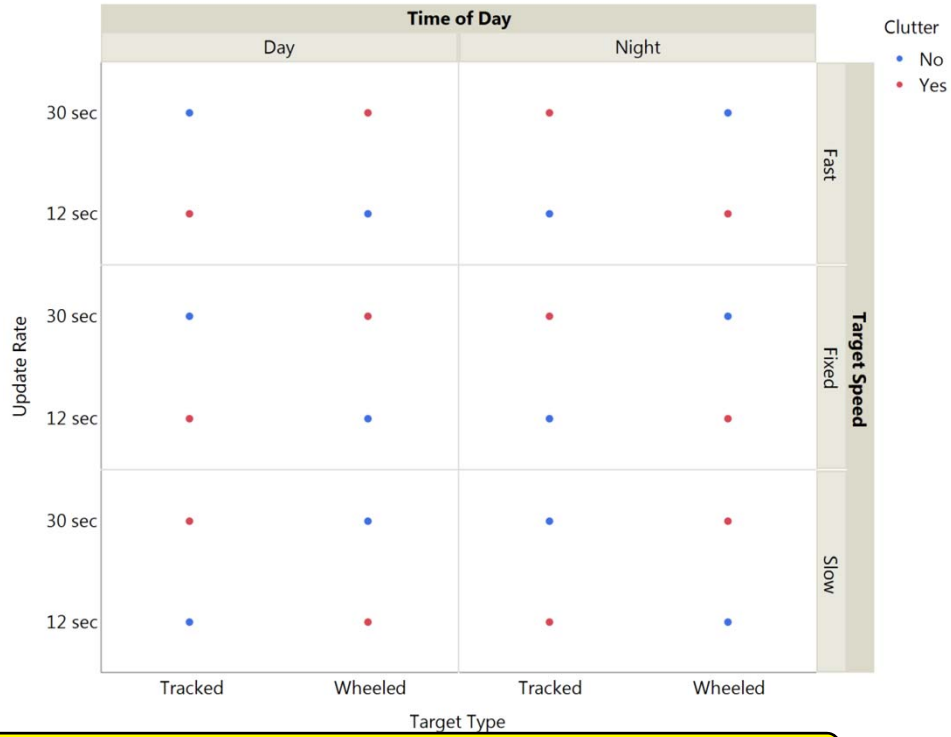
Another Perspective: Why Design for Two-Factor Interactions?

- **Interactions not only provide us with more flexibility in analyzing the data, but also provide an indication of the coverage of the operational space**
- **Small Diameter Bomb II Simplified Normal Attack Example**
- **Factors:**
  - Time of Day (Day/Night)
  - Update Rate (12, 20 sec)
  - Target Type (Tracked/Wheeled)
  - Target Speed (Fixed/Slow/Fast)
  - Clutter (Yes/No)
- **Design for Main Effects Only**
  - 7 run minimum
  - 12 run design shown
  - Sparse coverage
  - Low power



Why Design for Two Factor Interactions?

- **Interactions not only provide us with more flexibility in analyzing the data, but also provide an indication of the coverage of the operational space**
- **Small Diameter Bomb II Simplified Normal Attack Example**
- **Factors:**
  - Time of Day (Day/Night)
  - Update Rate (12, 20 sec)
  - Target Type (Tracked/Wheeled)
  - Target Speed (Fixed/Slow/Fast)
  - Clutter (Yes/No)
- **Design for Two-Way Interactions**
  - 21 run minimum
  - 24 run design shown
  - More complete coverage
  - Adequate power



A full factorial design would require 48 test points

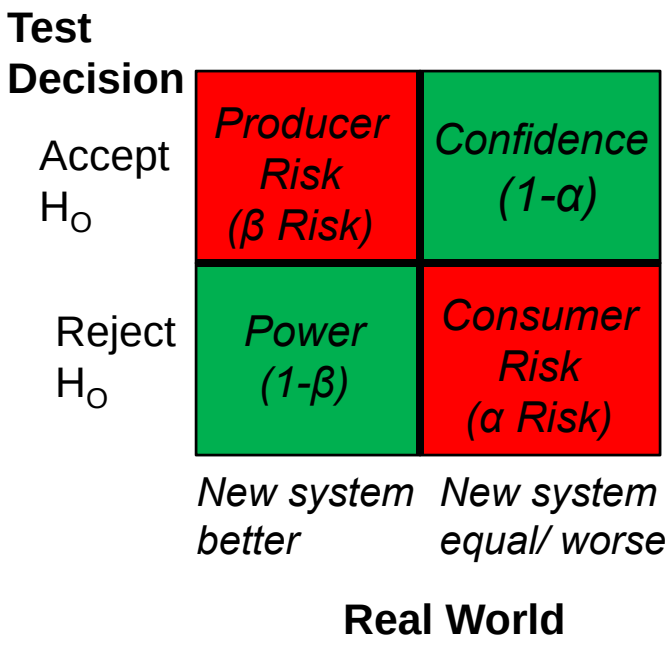
Power and Confidence

- **DOD 5000:** "acquire quality products that satisfy user needs with **measurable improvements** to mission capability and operational support"

- **Statistical Hypothesis Test:**
  - H_O: New system equal to or worse than the legacy system
  - H_A: New system **better** than the legacy system

- **Confidence**
  - Confidence Level – the probability we make the right decision based on the test data if the new hypothesis is true. In this case confidence tells us the probability that a test will conclude a systems is bad, when it truly is a bad system.

- **Power**
  - Similar to confidence level, power is the probability that we will make the right decision under one version of the alternative hypothesis. In this case power is the probability that a test will conclude a system is good, when it truly is a good system.



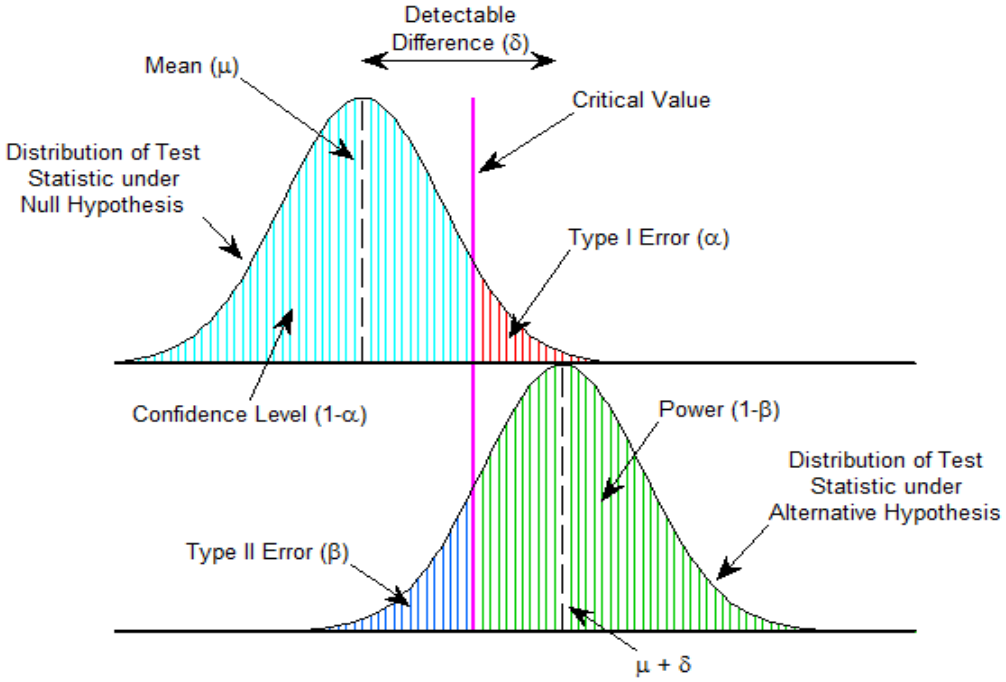
We need to understand risk.

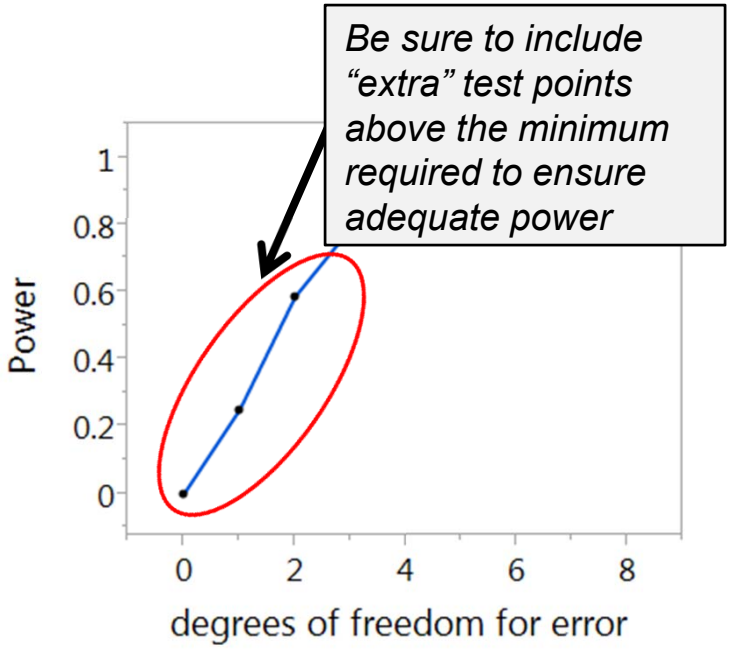
So what are confidence and power?

- **Confidence and power are only meaningful in the context of hypothesis test**
- **Confidence describes the risk of "False Positive" (Type I Error)**
  - Associated with the null hypothesis
  - What risk are we willing to accept of falsely rejecting the null hypothesis?
- **Power describes the risk of a "False Negative" (Type II Error)**
  - Associated with the alternative hypothesis
  - What risk are we willing to accept of falsely failing to reject the null hypothesis?
- **In designed experiments the hypothesis we are testing is:**
  - Null hypothesis: Factor has no effect on system performance
  - Alternative hypothesis: Factor does effect system performance

Power Analysis Overview

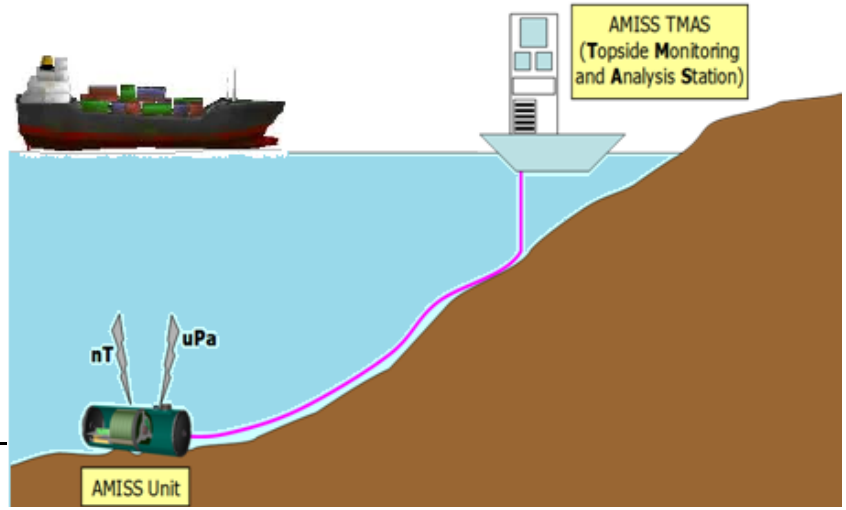
- **Power is a function of:**
  - Detectable difference
  - Variance
  - Confidence level (Typically we set confidence and calculate power)
  - Number of test points
  - Test points not committed to estimating model terms (error degrees of freedom)





How Much Testing is Enough?
Recall: Mine Susceptibility Testing Example

- **Goal:**
  - Develop an adequate test to assess the susceptibility of a cargo ship against a variety of mine types using the Advanced Mine Simulation System (AMISS).
- **Responses:**
  - Magnetic signature, acoustic signature, pressure
  - Slant range at simulated detonation
- **Factors:**
  - Speed, range, degaussing system status
- **Other considerations:**
  - Water depth
  - Ship direction



How much testing is enough? Power and Confidence

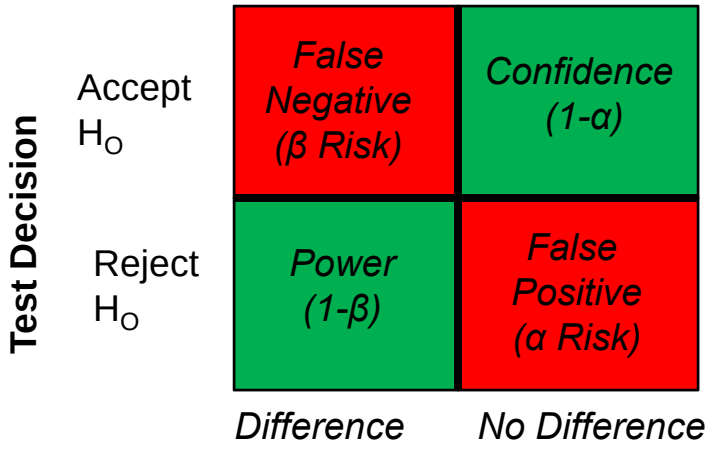
- **Power and confidence are only meaningful in the context of a hypothesis test!**
- **Statistical hypotheses:**

H_0: *Detonation slant range is the same with and without degaussing*

H_1: *Detonation slant range differs when degaussing is employed*

$$H_0: \mu_D = \mu_{ND}$$
$$H_1: \mu_D \neq \mu_{ND}$$

- **Power is the probability that we conclude that the degaussing system makes a difference when it truly does have an effect.**
- **Similarly, power can be calculated for any other factor or model term**

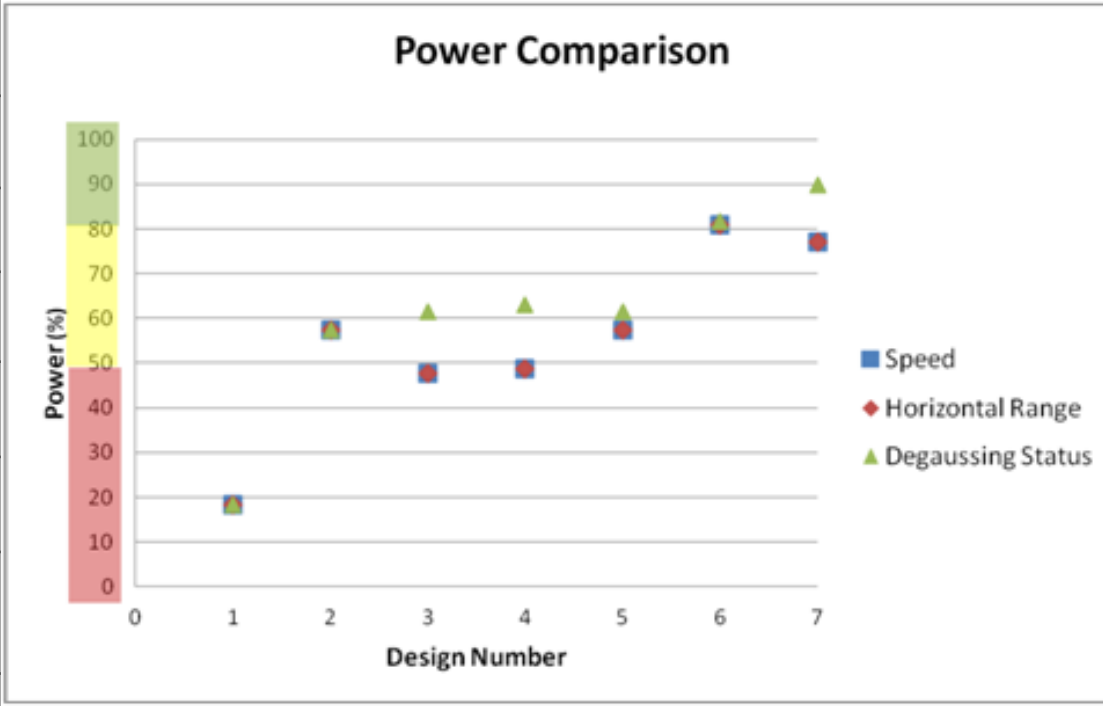


Power and confidence allow us to understand risk

Test Design Comparison: Statistical Power

- **Compared several statistical designs**
  - Recommended a replicated central composite design with 28 runs
  - Power calculations are for effects of one standard deviation at the 90% confidence level

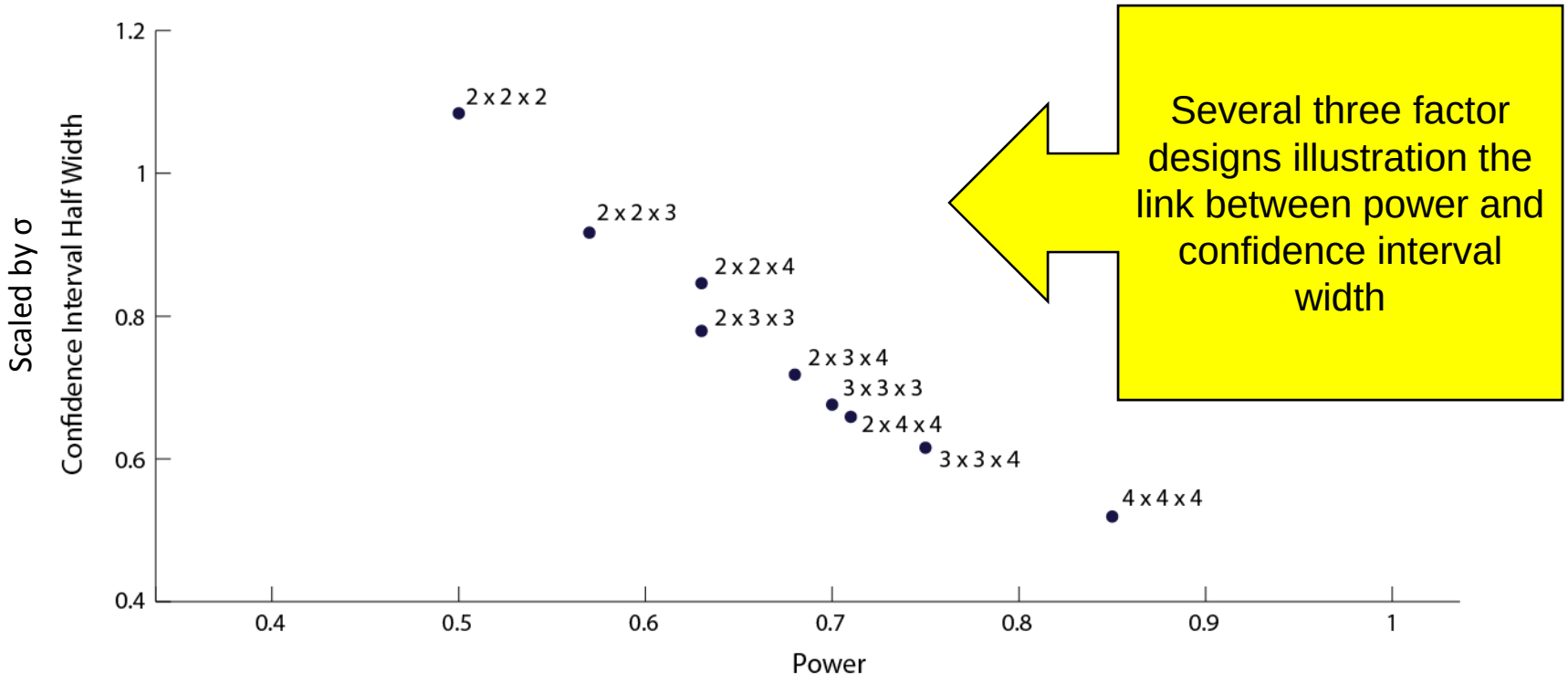
	Design Type	Number of Runs
1	Full Factorial (2-level)	8
2	Full Factorial (2-level) replicated	16
3	General Factorial (3x3x2)	18
4	Central Composite Design	18
5	Central Composite Design (replicated center point)	20
6	Central composite Design with replicated factorial points (Large CCD)	28
7	Replicated General Factorial	36



Statistical power provides an objective measure of how much testing is enough

IDA The Relationship between Power and Prediction

- **In operational testing, we are often most concerned with post test predictions and the width of our interval estimates**
- **Power provides a strong indication of how wide the confidence intervals when reporting results**



A Final Caution: Factor Power vs. One Sample Power

- **If characterization is the goal, then avoid one-sample hypothesis tests on average performance, they can be highly misleading**
- **Stryker Mobile Gun System hypothetical test designs**

		Mission	Attack				Defend			
Illum	OPFOR	Terrain	Urban	Mixed	Forest	Desert	Urban	Mixed	Forest	Desert
Day	Low		1	1	1	1	1	1	1	1
Day	Med		1	1	1	1	1	1	1	1
Day	High		1	1	1	1	1	1	1	1
Night	Low		1	1	1	1	1	1	1	1
Night	Med		1	1	1	1	1	1	1	1
Night	High		1	1	1	1	1	1	1	1

One Sample Power
99.5%

Factor Power	
Illum	98.4%
OPFOR	88.7%
Terrain	75.3%
Type	98.4%

vs

		Mission	Attack				Defend			
Illum	OPFOR	Terrain	Urban	Mixed	Forest	Desert	Urban	Mixed	Forest	Desert
Day	Low					8				
Day	Med				8					
Day	High			8						
Night	Low					8				
Night	Med				4	4				
Night	High			4		4				

One Sample Power
99.5%

Factor Power	
Illum	95.8%
OPFOR	47.5%
Terrain	41.3%
Type	N/A

Collinearity/Correlation Coefficients

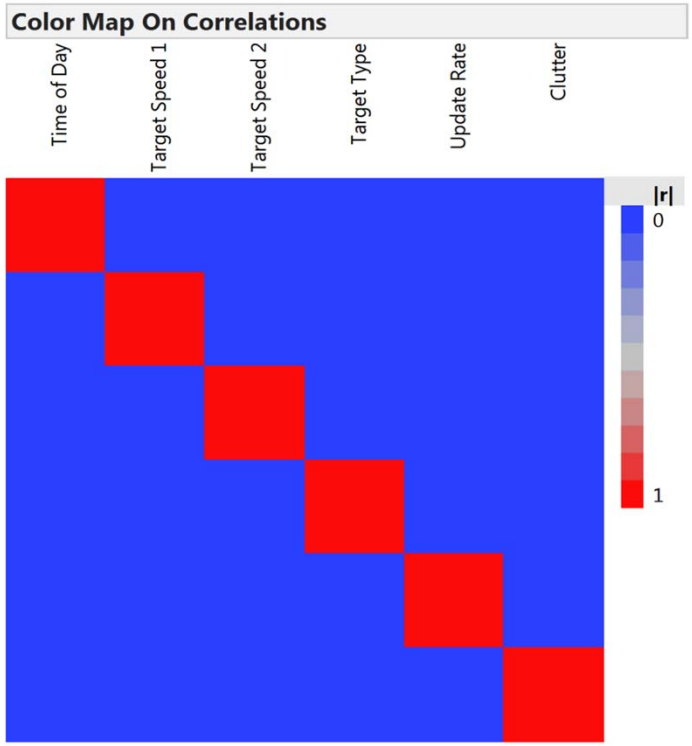
- **Two or more factors are consider collinear if they move together linearly (as one increases, so does the other)**
- **A well designed experiment minimizes the amount of collinearity between factors**

*Collinearity between factors **decreases** the power of a DOE and increases CI width*

- **Ideally, operational tests should be designed to support at least all main effects and two way interactions.**
  - When there are a large number of factors, it is often not possible to design operational tests to this standard
- **Correlation plots allow us to understand the tradeoffs in modeling**

Correlation Coefficients - No Correlation

- **Small Diameter Bomb II Normal Attack Example**
  - 12 Run Main Effects Only Design

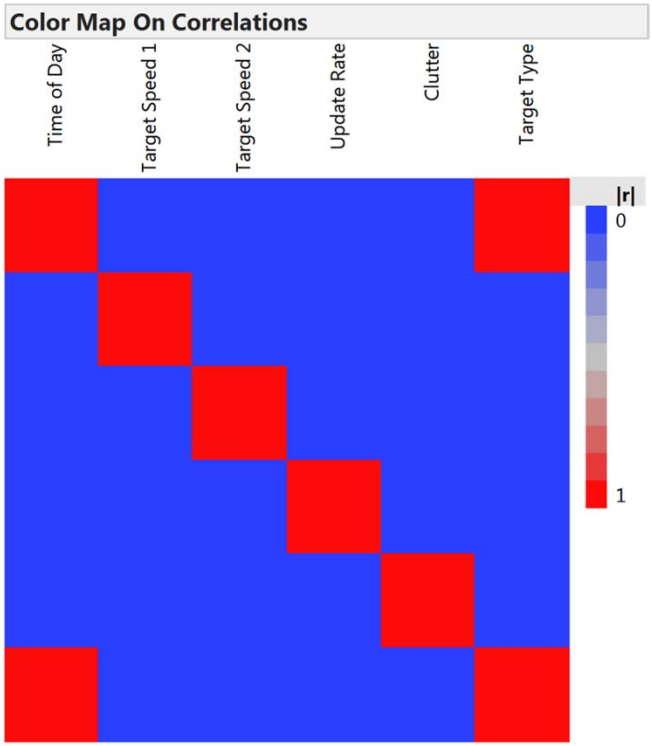


Time of Day	Target Speed	Target Type	Update Rate	Clutter
Day	Fast	Tracked	30	Y
Day	Fast	Wheeled	12	N
Day	Fixed	Tracked	12	N
Day	Fixed	Wheeled	30	N
Day	Slow	Tracked	12	Y
Day	Slow	Wheeled	30	Y
Night	Fast	Tracked	30	N
Night	Fast	Wheeled	12	Y
Night	Fixed	Tracked	30	Y
Night	Fixed	Wheeled	12	Y
Night	Slow	Tracked	12	N
Night	Slow	Wheeled	30	N

- **Even though this design is not a full factorial the main effects are all uncorrelated**
  - Blue is perfectly uncorrelated
  - Red is perfectly (100%) correlated

Correlation Coefficients – 100% Correlation

- **Small Diameter Bomb II Normal Attack Example**
  - 12 Run Main Effects Only Design

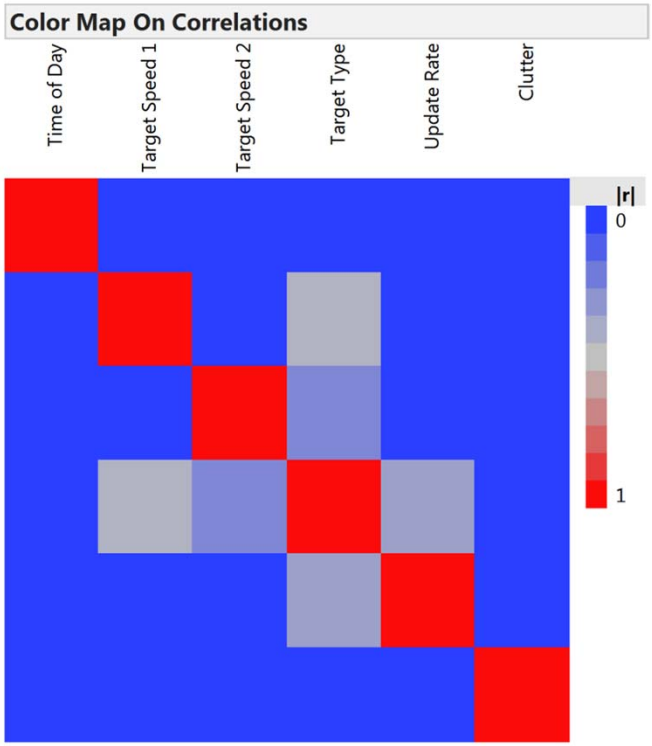


Time of Day	Target Speed	Target Type	Update Rate	Clutter
Day	Fast	Wheeled	30	Y
Day	Fast	Wheeled	12	N
Day	Fixed	Wheeled	12	N
Day	Fixed	Wheeled	30	N
Day	Slow	Wheeled	12	Y
Day	Slow	Wheeled	30	Y
Night	Fast	Tracked	30	N
Night	Fast	Tracked	12	Y
Night	Fixed	Tracked	30	Y
Night	Fixed	Tracked	12	Y
Night	Slow	Tracked	12	N
Night	Slow	Tracked	30	N

- **Target type is perfectly correlated with time of day!**
  - Blue is perfectly uncorrelated
  - Red is perfectly (100%) correlated

Correlation Coefficients – Some Correlation

- **Small Diameter Bomb II Normal Attack Example**
  - 12 Run Main Effects Only Design

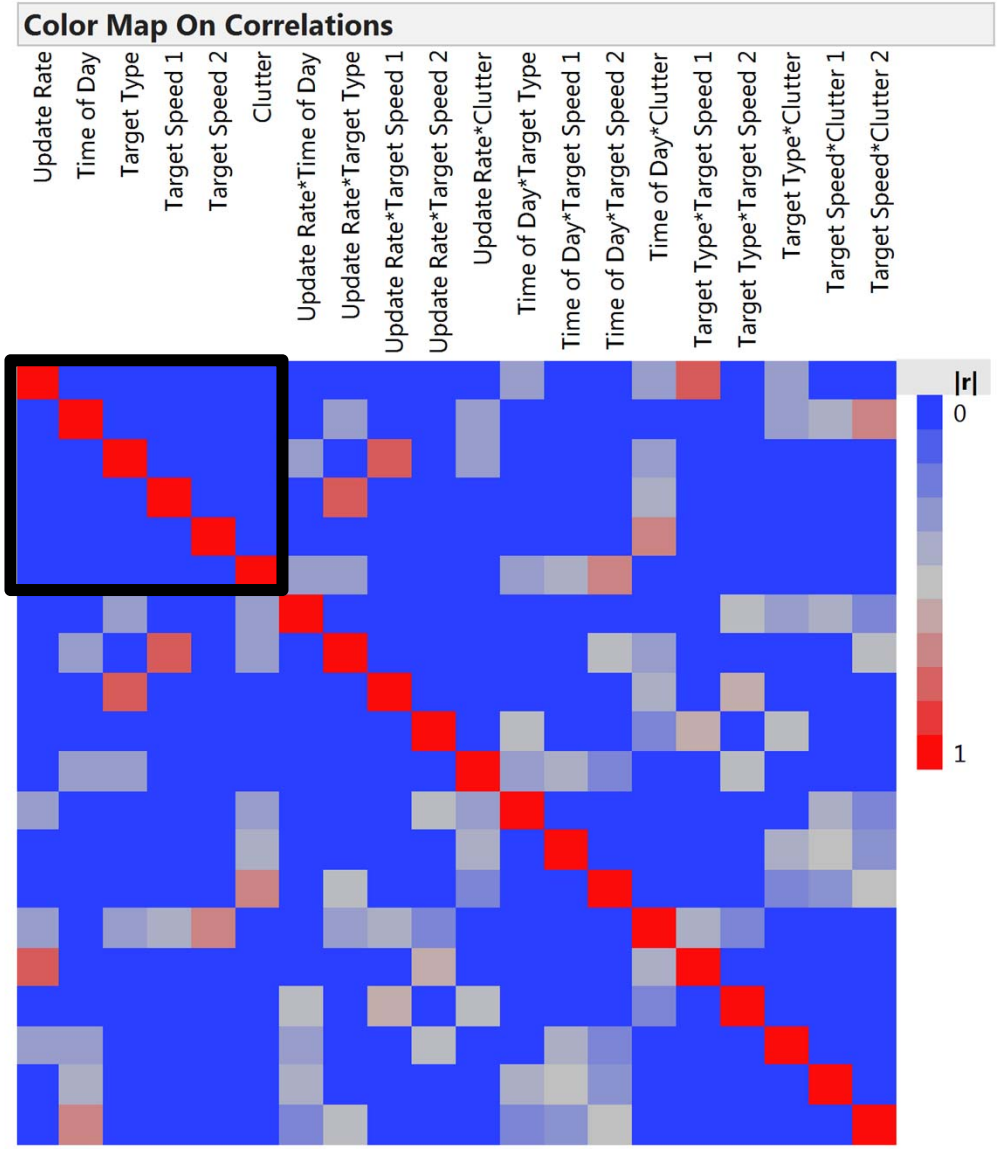


Time of Day	Target Speed	Target Type	Update Rate	Clutter
Day	Fast	Wheeled	30	Y
Day	Fast	Wheeled	12	N
Day	Fixed	Tracked	12	N
Day	Fixed	Wheeled	30	N
Day	Slow	Tracked	12	Y
Day	Slow	Wheeled	30	Y
Night	Fast	Wheeled	30	N
Night	Fast	Wheeled	12	Y
Night	Fixed	Tracked	30	Y
Night	Fixed	Wheeled	12	Y
Night	Slow	Tracked	12	N
Night	Slow	Wheeled	30	N

- **Practical constraints can introduce acceptable correlations**
  - e.g., Only wheeled vehicles can move fast

Model Supported and Correlation

- **Ideally, operational tests should be designed to support at least all main effects and two way interactions.**
  - When there are a large number of factors, it is often not possible to design operational tests to this standard
- **Correlation plots allow us to understand the tradeoffs in modeling**
- **Recall Original Small Diameter Bomb II Normal Attack Example – 12 Run Main Effects Only Design**
  - While correlation was zero between all main effects there are correlations greater than zero with two-way interactions



TEMP and Test Plan Review: Design Evaluation

1. **Overall Design Approach**
   - Is the test size proposed in the experimental design reasonable? Is it consistent with the resources section?
   - Are all the important factors included within the design?
2. **Model Supported**
   - Does the model supported at least a two-factor interaction model?
     - » If not, are the most important two-factor interactions estimable?
   - Are quadratic effects (or at least center points) included for continuous factors?
3. **Power**
   - Is power calculated for the primary response variables?
   - Is there high power for main effects?
     - » Note, there is no DOT&E rule for what constitutes “high power”
     - » Power calculations should always be based on expected effect size and estimated variance.
     - » However, in cases where no estimate is possible, historical data analysis have shown us that good rules of thumb are:
       - Confidence level = 95%, signal to noise ratio = 2
       - Confidence level = 80%, signal to noise ratio = 1
       - These guidelines should only be used as a first “sanity check”

TEMP and Test Plan Review: Design Evaluation

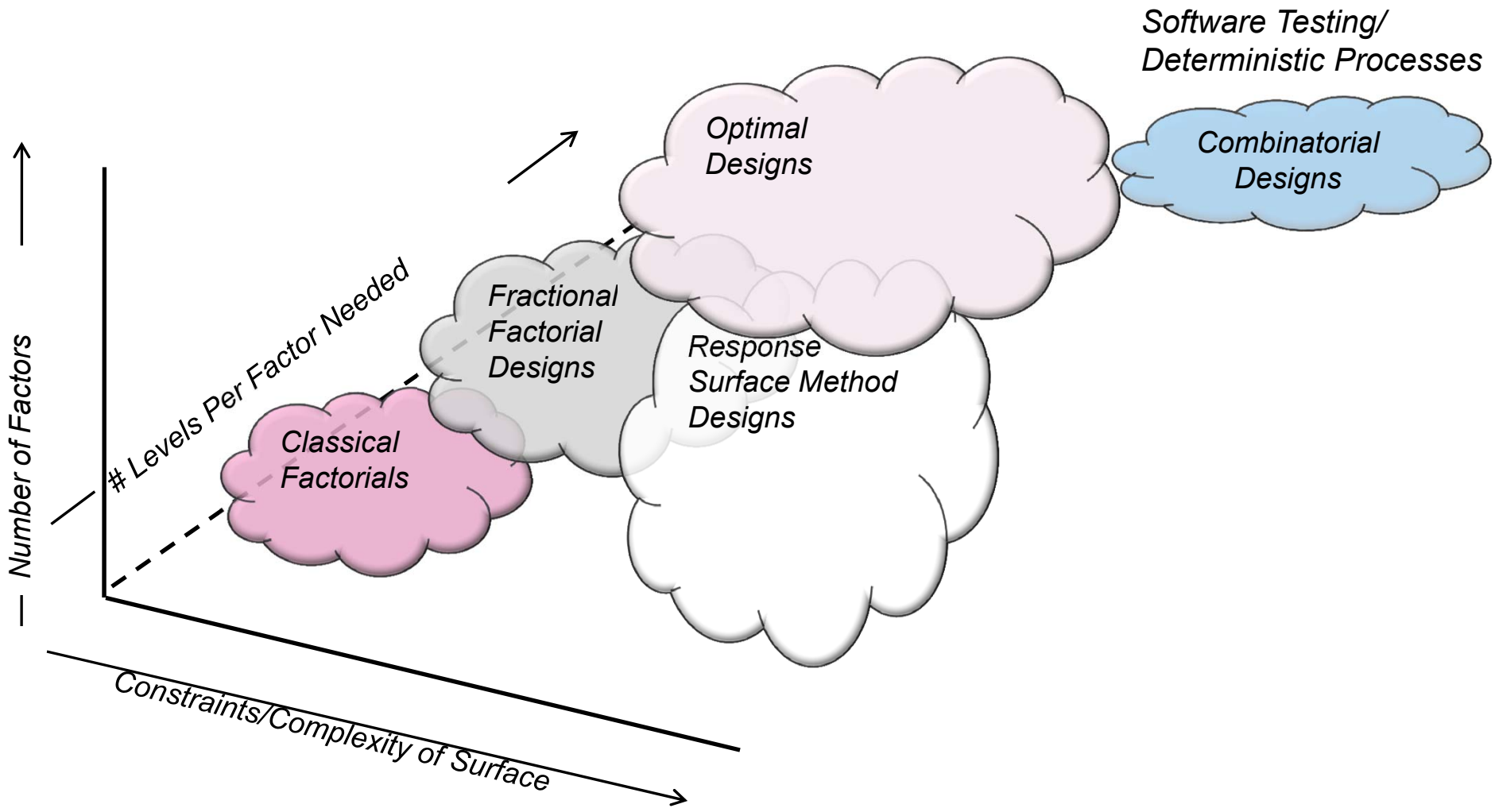
3. Power

- What is the power for interaction and higher order term effects?
  - » It is often reasonable to accept lower power for these terms
- What is the sensitivity of the power to the final analysis model?
  - » That is, if none of the interaction effects are significant, how does the power change for main effects?
- Note: power calculations should only need to be provided for the few primary response variables identified for the test, not every measure in the TEMP/Test Plan
  - » If the one primary response variable is pass/fail (binary) and another is continuous, separate power calculations should be provided

4. Correlation

- Is there low correlation (< 0.5) between all anticipated model terms?
- If not, why is the correlation structure acceptable?

Design Coverage



Common Design Mistakes

- **Failure to link goals, responses, factors, levels, and resourcing**
  - All elements may be present but there also needs to be a linkage
- **Failure to link analysis to the design**
  - Roll-up power calculations versus power calculations by factor
  - Power should always be reported for at least all main effects!
- **Elimination of factors or factors left uncontrolled, because "we can't afford that many factors in a design"**
  - Sparsity of effects
  - Fractional factorials, small response surface designs, and optimal designs can support a large number of factors.
- **Trying to build one design for the full operational test**

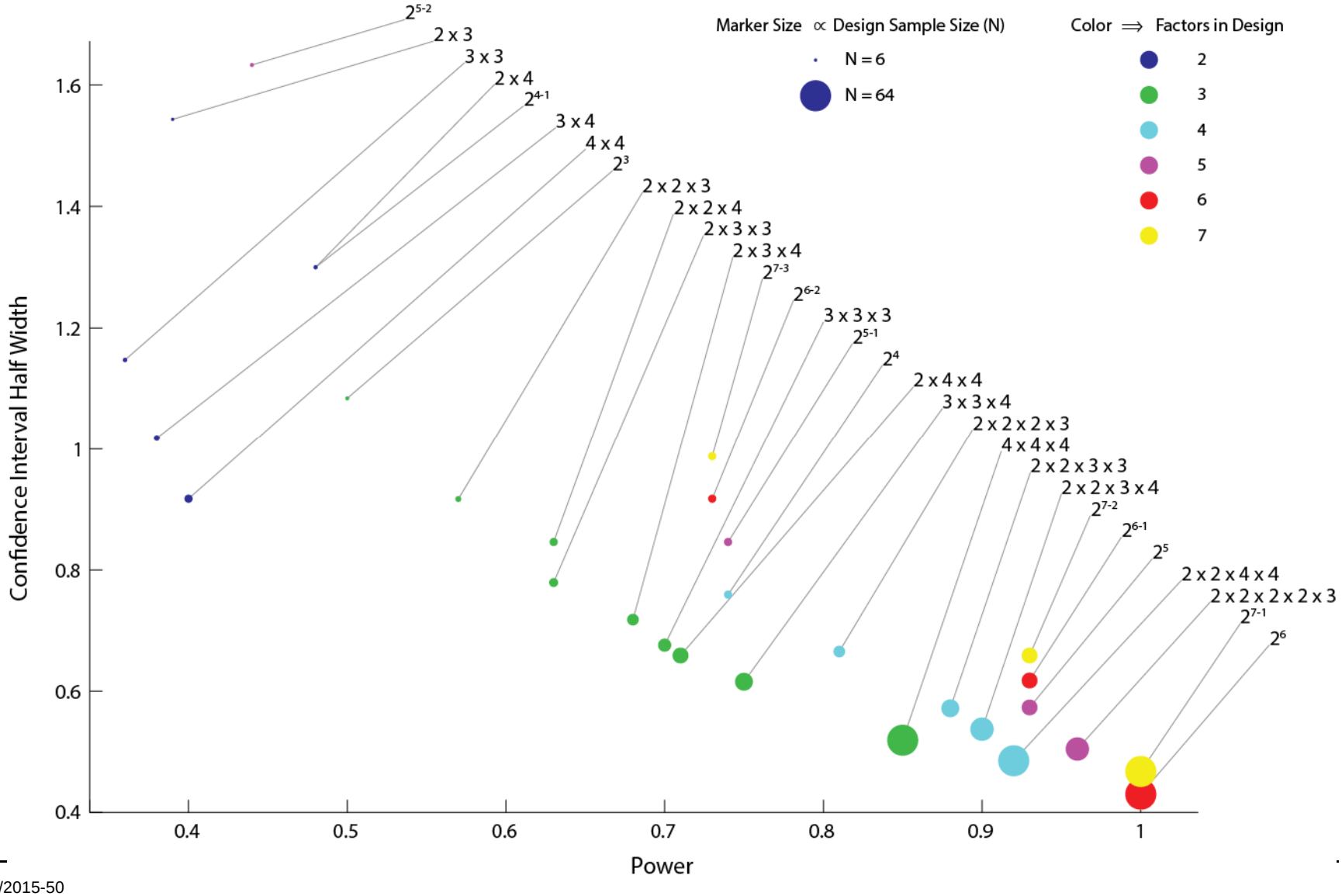
Key Takeaways: Experimental Design

- **There are many types of experimental designs**
  - Design choice depends on test objectives, number of factors/levels, and risk tolerance
- **Test designs should support characterization**
  - Characterization implies that we are interested in predictions across the design space
  - This typically requires designing for models that contain at least main effects and most two-factor interactions
  - Higher order terms improve predictions
- **Power is a useful metric for assessing test adequacy and selecting an appropriate design**
  - Other measures exist, correlation structures are useful tools for explaining test designs.
  - Roll-up power calculations are misleading and inappropriate for assessing designed experiments

Backup Material

Power versus Confidence Interval Half Width

for various factorial experiments



Common DOE Methods for Software

- **Factor Covering or Combinatorial Designs**
  - How to test as quickly as possible when the test space is large and made up of combinations of selections
- **Space Filling**
  - How to spread out test cases evenly when the test space is large and continuous
    - For example,
- **Both methods improve the chance of finding defects that are 'combinatorial' or 'regional'**

Software Test Design: Risk in Test

- **Traditional Textbooks of DOE focuses on statistical risk (probabilistic)**
- **Modern DOE includes design-methods for software, where non-statistical risks can be a primary (or even the only) focus**

Statistical-Risk

- **Outcomes may change for one set of inputs, the change is 'diffused' across the levels of the input factors**
- **The RISK: Data is confusing or not representative due to random chance**

Non-Statistical-Risk

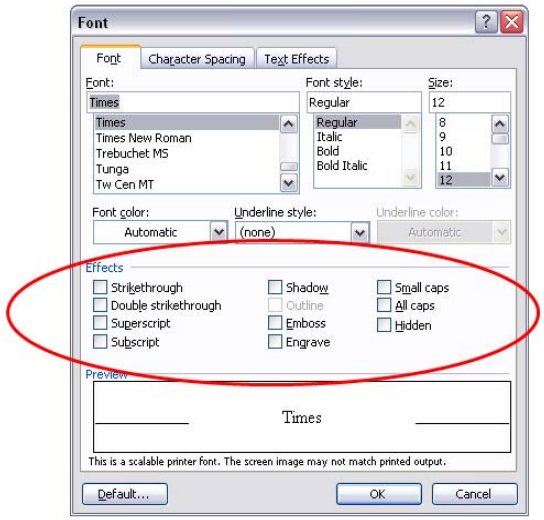
- **Outcomes may change little for one set of inputs, but may change unpredictably if the inputs are changed**
- **The RISK: Defects go undetected thanks to incomplete or inefficient coverage of the space**

In software testing the goal is to cover as much of the space as possible, this is done at the expense of being able to determine cause and effect relationships!

Combinatorial Designs

- **Combinatorial Test Designs are tests that cover a large number of combinations of factors extremely quickly, searching for problems**
  - Trade-off: we lose the ability to determine cause and effect, therefore process must be deterministic
  - Examples: bugs in software, link inoperability

- **Everyday example:**

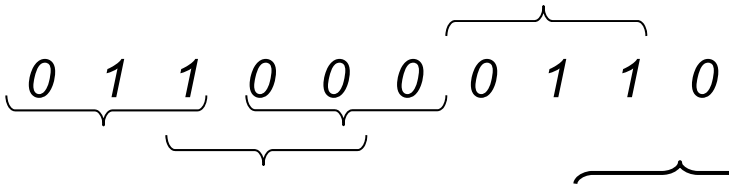


How many tests?

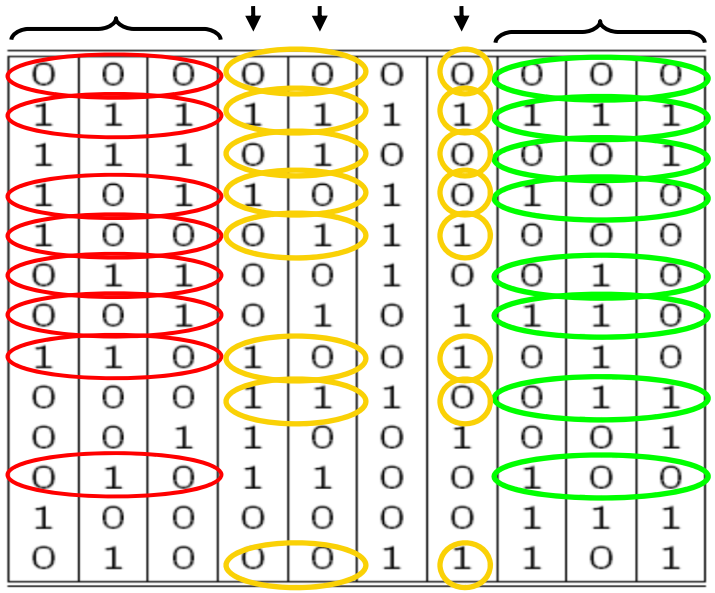
- There are 10 effects, each can be on or off
- All combinations equals 2^{10} = 1,024 tests
- What if our budget is too limited for these tests?
  - Main effects are easy – 10 tests
  - But what about interactions?
    - 90 two-way interactions
    - 120 three-way interactions

Combinatorial Designs

- **How can we cover all 120 three-way interactions?**
- **Since we can pack 3 triples into each test, we need no more than 40 tests.**
- **Each test exercises many triples:**



- **Each row is several simultaneous tests**
- **Finding combinatorial designs is difficult a process.**
  - Requires computer software
    - NIST
    - Hexawise
    - JMP 12 Pro



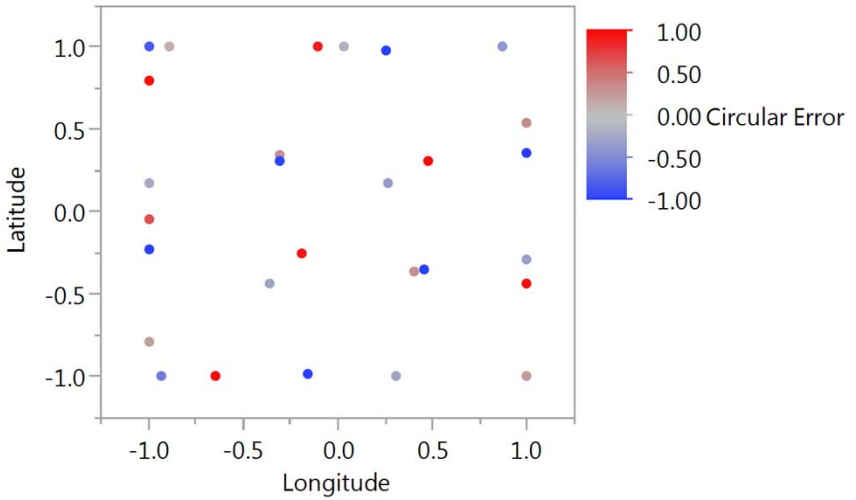
Factor Covering

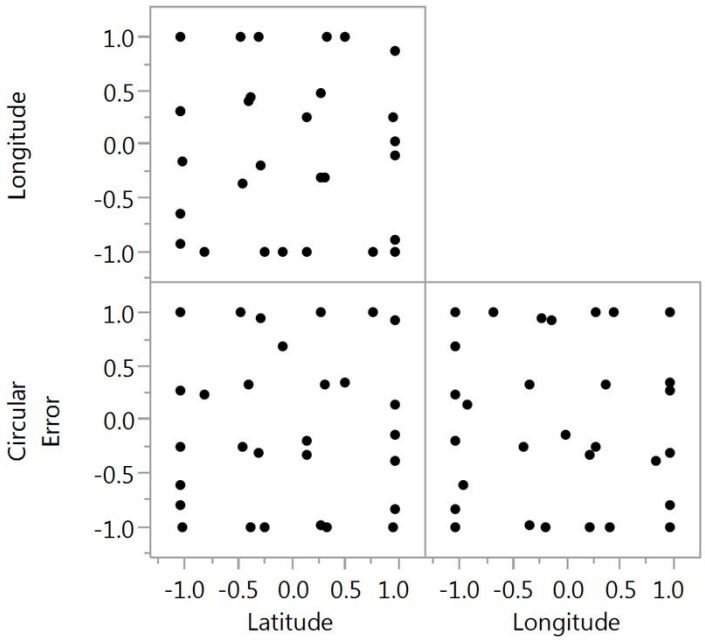
Box 1	Box 2	Box 3	Box 4	Box 5	Box 6	Box 7	Box 8	Box 9	Box 10
1	1	1	1	1	1	1	1	1	1
1	1	1	1	0	0	0	0	0	0
1	0	0	0	1	1	1	0	0	0
0	1	0	0	1	0	0	1	1	0
0	0	1	0	0	1	0	1	0	1
0	0	0	1	0	0	1	0	1	1

It takes FIVE cases to cover all pairs for FOUR factors (2 levels)
We can extend into TEN dimensions with only ONE more case…

Space Filling

- **Space Filling is an efficient way to search or cover continuous input spaces**
- **Space Filling algorithms spread out test points using tailored optimality criteria**
- **3 popular algorithms:**
  - Sphere-Packing
    - » Maximize the smallest distance between neighbors
    - » Effect: Moves points out to boundaries
  - Uniform
    - » Minimize discrepancy from a uniform distribution
    - » Effect: Spreads points within interior
  - Latin Hypercube
    - » Assign n congruent levels and minimize covariance
    - » Effect: Combination of the above





Summary of DOE for Software

- **There is a science to software system test**
- **The appropriateness of designs depends on if outcomes are deterministic vice probabilistic**
- **There ARE tools and techniques (we covered *some*) that have utility for software-intensive test design:**
  - Factor Covering for covering sub-configurations (categorical factors)
  - Space Filling for spanning regions (continuous factors)